UG–745 **BMS-03**

B.Sc. DEGREE EXAMINATION – JUNE 2006.

First Year

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Solve : $xp^2 - 2yp + x = 0$.

2. Solve : $(D^2 + 3D + 2)\, y = x^2$.

3. Solve : $(D^2 - 2D + 4)\, y = e^x . \cos x$.

4. Form a partial differential equation by eliminating the function of ϕ from

$$\phi\,(x + y + z,\ x^2 + y^2 - z^2) = 0.$$

5. Solve : $(mz - ny)\,p + (nx - lz)\,q = ly - mx$.

6. Solve : $z = px + qy + c\sqrt{1 + p^2 + q^2}$.

7. Prove that $L\,[t^n] = \dfrac{\Gamma(n+1)}{s^{n+1}}$ and hence find $L\,[t^{\frac{1}{2}}]$.

8. Find $L^{-1}\left[\dfrac{s+2}{(s^2 + 4s + 5)^2}\right]$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Solve : $(2x+1)^2\ y'' - 2(2x+1)y' - 12y = 6x$.

10. Solve : $(4D+2)x + (9D+31)y = e^t$

$(3D+1)x + (7D+24)y = 3$.

11. Solve : $\frac{d^2y}{dx^2} + 4y = \operatorname{cosec}(2x)$ by the method of variation of parameters.

12. Verify the condition of integrability in the equation $(y+z)\,dx + (z+x)\,dy + (x+y)\,dz = 0$ and solve it.

13. Solve : $p + 3q = 5z + \tan(y - 3x)$.

14. Solve : $p^3 + q^3 = 27z$.

15. (a) Find $L[(f(t)]$ where

$$f(t) = 0 \text{ when } 0 < t < 2$$
$$= 3 \text{ when } t > 2.$$

(b) Find $L^{-1}\left[\frac{s^2}{(s-1)^3}\right]$.

16. Using Laplace transform, solve the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$ given that $y = \frac{dy}{dt} = 0$ when $t = 0$.

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UG–467 **BMS-04**

B.Sc. DEGREE EXAMINATION
JANUARY 2009.

(AY – 2005-06 and CY – 2006 batches only)

Second Year

Mathematics

MODERN ALGEBRA

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. If the Function $f : R \to R$ is given by $f(x) = x^2$ and $g : R \to R$ is given by $g(x) = \sin x$, find $(g \circ f)(x)$ and $(f \circ g)(x)$ and show that they are not equal.

2. If H and K are subgroups of a given group G, then prove that $H \cap K$ is also a subgroup of G.

3. How many generators are there for a cyclic group of order 10?

4. If n is any integer and $(a,n)=1$ then prove that $a^{\phi(n)} \equiv 1 \pmod{n}$.

5. Prove that every subgroup of an abelian group is a normal subgroup.

6. If $f : G \to G'$ is a homomorphism then show that f is one to one if and only if $\ker f = \{e\}$.

7. Prove that the set of all matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ where $a, b \in R$ is a ring under matrix addition and multiplication.

8. If R is a commutative ring with identity then prove that R is an integral domain if and only if cancellation law is valid in R.

SECTION B — $(5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. If A and B are two subgroups of a group G then prove that AB is a subgroup of G if and only if $AB = BA$.

10. If G is a group and $a \in G$ then prove that the order of a is the same as the order of the cyclic group generated by a.

11. Prove that a group G has no proper subgroups if and only if it is a cyclic of prime order.

12. If a group G has exactly one subgroup H of given order then prove that H is a normal subgroup of G.

13. Prove that any finite group is isomorphic to a group of permutations.

14. State and prove the fundamental theorem of homomorphism.

15. Prove that any finite integral domain is a field.

16. Prove that any integral domain D can be embedded in a field F and every element of F can be expressed as a quotient of two elements of D.

UG–468 **BMS-05**

B.Sc. DEGREE EXAMINATION
JANUARY 2009.

(AY – 2005-06 and CY – 2006 batches only)

Second Year

Mathematics

STATISTICS

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Compute the value of Geometric mean :

Marks :	10	20	25	40	50
Frequency :	20	30	50	15	5

2. Compute coefficient of correlation for the following data :

A	9	8	7	6	5	4	3	2	1
B	15	16	14	13	11	12	10	8	9

3. Obtain first 4 moments about mean for the following data :

X :	2	3	4	5	6
f :	1	3	7	3	1

4. Construct index numbers of Price from the following data :

	2006		2007	
Commodity	Price	Quantity	Price	Quantity
A	2	8	4	6
B	5	10	6	5
C	4	14	5	10
D	2	19	2	13

5. One card is drawn from a standard pack of 52. What is the probability that it is either a king or a queen?

6. For the Binomial distribution the mean is 6 and S.D. is $\sqrt{2}$. Obtain n, p and q.

7. A man buys 50 electric bulbs of 'Philips' and 50 electric bulbs of 'HMT'. He finds that 'Philips' bulbs give an average life of 1,500 hours with a standard deviation of 60 hours and 'HMT' bulbs gave an average life of 1,512 hours with a standard deviation of 80 hours. Is there a significant difference in the main life of the two makes of bulbs?

8. In a sample of 8 observations, the sum of squared deviations of items from the mean was 84.4. In another sample of 10 observations, the value was found to be 102.6. Test whether the difference is significant at 5% level.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Calculate quartile deviation and the coefficient of quartile deviation from the following data :

Wages in Rupees Per week :	less than 35	35–37	38–40	41–43	Over 43
Number of wage earners :	14	62	99	18	7

10. Calculate coefficient of skewness :

X :	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5
f :	28	42	54	108	129	61	45	33

11. From the following data obtain the two regression equations :

X :	1	2	3	4	5	6	7	8	9
Y :	9	8	10	12	11	13	14	16	15

12. Using Newton's formula find the annual premium at the age of 33 from the following data :

Age in years :	24	28	32	36	40
Annual premium in Rs :	28.06	30.19	32.75	34.94	40

13. From the following data compute price index by supplying weighted average of price method using arithmetic mean

Commodity	P_0 (Rs.)	Q_0	P_1 (Rs.)
sugar	3.0	20 kg	4.0
flour	1.5	40 kg	1.6
milk	1.0	10 lt.	1.5

14. A box contains 3 red and 7 white balls. One ball is drawn at random and in its place a ball of the other colour is put in the box. Now one ball is drawn at random from the box. Find the probability that it is red.

15. Assume the mean height of solidiers to be 68.22 inches with a variance of 10.8 inches. How many solidiers in a regiment of 1000 would you expect to be over six feet tall?

16. In an experiment on immunization of cattle from tuberculosis, the following results were obtained

	Affected	Not affected
Inoculated	12	26
Not inoculated	16	6

Calculate χ^2 and discuss the effect of vaccine in controlling susceptibility to tuberculosis.

UG–469 **BMS-06**

B.Sc. DEGREE EXAMINATION
JANUARY 2009.

(AY – 2005-06 and CY – 2006 batches only)

Second Year

Mathematics

MECHANICS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

Each question carries 5 marks.

1. Determine the magnitude and direction of the resultant of two given forces with a common point of application.

2. $ABCDEF$ is regular hexagon. At A, for as represented in magnitude and direction by $\overrightarrow{AB}, 2\overrightarrow{AC}, 3\overrightarrow{AD}, 4\overrightarrow{AE}$ and $5\overrightarrow{AF}$ act. Show that their resultant is of magnitude $\sqrt{351}AB$ and is inclined at the angle $\tan^{-1}\left(7/\sqrt{3}\right)$ to AB.

3. Obtain the components of a given force along two specified directions.

4. Find the greatest height attained by the projectile in a parabolic path.

5. If a particle moves along a circle with uniform angular velocity, show that its projection on any diameter executes a simple harmonic motion.

6. Find the components of acceleration of a particle in tangential and normal directions.

7. A point P describes with a constant angular velocity about O the equiangular spiral $r = ae^{\theta}$, O being the pole of the spiral. Obtain the radial and transverse acceleration of P.

8. A partical describes the orbit $\frac{l}{r} = 1 + e\cos\theta$, under a central force, the pole being the centre. Find the law of force.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

9. State and prove polygon law of forces.

10. State and prove Lame's theorem.

11. State and prove Varignm' theorem on moments.

12. Two like parallel forces P and Q $(P > Q)$ act at A and B respectively. If the magnitudes of the forces are inter-changed show that the point of application of the resultant on AB will be displaced through the distance $\frac{P-Q}{P+Q} \cdot AB$.

13. Find the path of a projectile with initial velocity u and angle of projection α.

14. Find the velocity of two smooth spheres after a direct impact between them.

15. Obtain the resultant of two simple harmonic motions of the same period in the same straight lines.

16. Derive the pedal equation of the central orbit.

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UG–470	**BMS–07**

B.Sc. DEGREE EXAMINATION –
JANUARY 2009.

(AY – 2005-06 and CY – 2006 batches only)

Third Year

Mathematics

REAL AND COMPLEX ANALYSIS

Time : 3 hours　　　　Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

1. Prove that in any metric space, the union of any family of open sets is open.

2. Define a convergent sequence. Show that for a convergent sequence (x_n) the limit is unique.

3. If (M, d) is a metric space and $a \in M$, prove that the functions $f : M \to R$ defined by $f(x) = d(x, a)$ is continuous.

4. Show that any discrete metric space with more than one point is disconnected.

5. Show that the function $f(z) = \frac{\bar{z}}{z}$ does not have a limit as $z \to 0$.

6. Find the bilinear transformation which maps the points $z = -1, 1, \infty$ respectively on $w = -i, -1, i$.

7. State and prove Lioville's theorem.

8. Define residue of $f(z)$ at an isolated singularity calculate the residues of $\frac{z+1}{z^2 - 2z}$ at its poles.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

9. State and prove Minkowski's inequality.

10. Show that l_2 is complete.

11. Prove that f is continuous if and only if inverse image of every open set is open.

12. Show that any compact subset A of a metric space (M, d) is closed.

13. Show that the points z_1 and z_2 are reflection points for the line $\overline{\alpha}z + \alpha\overline{z} + \beta = 0$ if and only if $\overline{\alpha}z_1 + \alpha\overline{z}_2 + \beta = 0$.

14. State and prove a sufficient condition for differentiability of complex valued function.

15. State and prove Taylor's theorem.

16. State and prove Rouche's theorem.

UG–471	**BMS–08**

B.Sc. DEGREE EXAMINATION –
JANUARY 2009.

Third Year

(A.Y. 2005–06 and C.Y. 2006 batches only)

Mathematics

LINEAR ALGEBRA AND NUMBER SYSTEM

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that the set of complex numbers C is a vector space over the field **R**.

2. Define inner product space. Give an example.

3. Compute the inverse of the matrix $\begin{pmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$.

4. Verify Cayley-Hamilton theorem for the matrix $\begin{pmatrix} 2 & -1 & 1 \\ 1 & 0 & 2 \\ 3 & -1 & 3 \end{pmatrix}$.

5. Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{pmatrix}$.

6. Find the smallest number with 18 divisors.

7. Show that $x^5 - x$ is divisible by 30.

8. Show that $7^{2n} + 16n - 1 \equiv 0 \pmod{64}$.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. Let V be a vector space over a field F. Let $S = \{v_1, v_2, \cdots, v_n\} \subseteq V$. Prove that the following are equivalent

(a) S is a basis for V.

(b) S is a Maximal linear independent set

(c) S is a minimal generating set.

10. If V is a finite dimensional vector space over a field F and W is a subspace of V, prove that $\dim \frac{V}{W} = \dim V - \dim W$.

11. Apply Gram-Schmidt process to construct orthonormal basis from the basis {(1, 0, 1), (1, 3, 1), (3, 2, 1)}.

12. Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$.

13. Reduce the matrix $\begin{pmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{pmatrix}$ to its normal form.

14. Define Eulers ϕ-function. Find the value $\phi(N)$, if $N = p^a q^b r^c \cdots$ where $p, q, r, \cdots$ are all primes and $a, b, c, \cdots$ are integers.

15. State and prove Fermat's theorem.

16. Show that $28! + 233 \equiv 0 \pmod{899}$.

UG–472 **BMS–09**

B.Sc. DEGREE EXAMINATION –
JANUARY 2009.

(AY – 2005-06 and CY – 2006 batches only)

Third Year

Mathematics

LINEAR PROGRAMMING AND OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Write the following linear programming problem in standard form.

Minimize $Z = 2x_1 - 3x_2 + x_3$

Subject to
$$-x_1 + 3x_2 \leq -5$$
$$x_1 + 2x_2 + x_3 \leq 6$$
$$x_1 + x_2 + x_3 \geq -8$$
$$x_1,\ x_2,\ x_3 \geq 0.$$

2. Explain the term artificial variables and its use in linear programming.

3. Explain the primal-dual relationship.

4. Give the mathematical formulation of an assignment problem. How does it differ from a transportation problem?

5. Obtain an initial basic feasible solution to the following transportation problem by using North West Corner Rule.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

6. Explain the following terms in inventory management.

   (a) Carrying cost

   (b) Shortage cost.

7. What is replacement? Describe some important replacement situations.

8. Explain : Minimax and Maximin principle used in the theory of games.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let $S = \{x_1, x_2 .... x_n\} \subseteq R^n$. Then prove that the set of all convex combinations of $\{x_1, x_2 .... x_n\}$ is a convex set in R^n.

10. Use simplex method to solve the following LPP :

Maximize $Z = 3x_1 + 2x_2$

Subject to
$$x_1 - x_2 \le 1$$
$$3x_1 - 2x_2 \le 6$$
$$x_1,\ x_2 \ge 0.$$

11. Solve the following assignment problem which minimises the total man hours.

$$\begin{array}{ccc} & & \begin{array}{cccc} A & B & C & D \end{array} \\ & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 18 & 26 & 17 & 11 \\ 13 & 28 & 14 & 26 \\ 38 & 19 & 18 & 15 \\ 19 & 26 & 24 & 10 \end{pmatrix} \end{array}$$

Jobs (rows 1–4)

12. Solve the following unbalanced transportation problem.

To

From						
	5	8	6	6	3	800
	4	7	7	6	5	500
	8	4	6	6	4	900
	400	400	500	400	800	

13. Neon lights in an industrial park are replaced at the rate of 100 units per day. The physical plant orders the neon lights periodically. It costs Rs. 100 to initiate a purchase order. A neon light kept in storage is estimated to cost about Re. 02 per day. The lead time between placing and receiving an order is 12 days. Determine the optimum inventory policy for ordering the neon lights.

14. The cost of a machine is Rs. 6,100 and its scrap value is only Rs. 100. The maintenance costs are found from experience to be :

Year :	1	2	3	4	5	6	7	8
Maintenance cost :	100	250	400	600	900	1250	1600	2000

When should the machine be replaced?

15. Find out the optimum strategies for the following 2×2 game without saddle point.

$$\begin{array}{cc} & \text{B} \\ \text{A} & \begin{pmatrix} 5 & 1 \\ 3 & 4 \end{pmatrix} \end{array}$$

16. In the model $(M/M/1):(N/FIFO)$ find P_n.

UG–473 **BMS–10**

B.Sc. DEGREE EXAMINATION – JANUARY 2009.

(AY – 2005-06 and CY – 2006 batches only)

Third Year

Mathematics

PROGRAMMING IN C AND C++

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. What is constant? What are the different types of constants in C? Explain in detail.

2. Describe the components of a format specifier in a printf function.

3. What is recursion? It is advantageous to use recursion in programming? Justify your answer with suitable example.

4. Explain string handling functions.

5. What is dynamic memory allocation? Explain how it is used to declare an array.

6. Explain bit fields.

7. Explain the differences between the access modes.

8. Write a program to find whether a given number is prime or not.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. Explain priority queues.

10. Explain the operations of linked list.

11. Write a program to read the name in one file and copy the contents to another file.

12. Write a suitable struct data type for an employee using nested struct.

13. Write the declaration for a pointer variable for an array of integers of size 3×4 and allocate a memory for that variable.

14. Explain extern storage class.

15. Write a program to find the value of ${}^{N}p_{r}$.

16. Explain switch statement.

UG–475 BMS–11/BMC–11

B.Sc. DEGREE EXAMINATION –
JANUARY 2009.

First Year

Mathematics/Mathematics with Computer Applications

ELEMENTS OF CALCULUS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. If $y = a\cos 5x + b\sin 5x$ show that $\frac{d^2y}{dx^2} + 25y = 0$.

2. Verify Euler's theorem for the function $u(x, y) = x^3 + y^3 + 3x^2y + 3xy^2$.

3. Find the envelope of the family of straight lines $x\cos\alpha + y\sin\alpha = p$ where α is the parameter.

4. Find the surface area of a sphere of radius a.

5. Evaluate $\int_0^{\pi/2} \int_0^a dr\, d\theta$.

6. Show that the sequence $\{a_n\}$ where $a_n = \frac{1}{n}$ for every $n \in N$ converges to 0.

7. Define

(a) Monotonic sequence

(b) Cauchy sequence.

8. Test the convergence of $\sum \sqrt{\frac{2n^2 + 3}{5n^3 + 7}}$.

SECTION B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)\, x y_{n+1} + (n^2 + 1)\, y_n = 0$.

10. Find the maximum and minimum values of $f(x, y) = 2(x^2 - y^2) - x^4 + y^4$.

11. Show that the radius of curvature of the curve $y^2 = a^2 \frac{(a - x)}{x}$ at $(a, 0)$ is $\frac{a}{2}$.

12. (a) Evaluate $\int_0^1 x\,(1-x)^{10}\,dx$.

(b) Prove that

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx .$$

13. Establish a reduction formula for $I_n = \int \cos^n x\,dx$ where $n \in N$ and hence find $\int_0^{\pi/2} \cos^6 x\,dx$.

14. If $\lim_{n \to \infty} a_n = a$ and $\lim_{n \to \infty} b_n = b$ then prove that $\lim_{n \to \infty} (a_n \pm b_n) = a \pm b$.

15. (a) Test the convergence of the series

$$\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \cdots \infty$$

(b) Show that a series $\sum_{n=1}^{\infty} u_n$ of positive terms either converges or diverges to ∞ but never oscillates.

16. Let Σu_n and Σv_n be the two given series of positive numbers such that $u_n < kv_n$ for every $n \in N$ where k is a positive number. Then prove that

(a) If Σv_n is convergent Σu_n is convergent

(b) If Σu_n is divergent then Σv_n is divergent.

UG–476 **BMS–12/ BMC-12**

B.Sc. DEGREE EXAMINATION — JANUARY, 2009.

First Year

(AY 2006-07 batch onwards)

Mathematics

TRIGONOMETRY, ANALYTICAL GEOMETRY OF THREE DIMENSIONS AND VECTOR CALCULUS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

Each question carries 5 marks.

1. Prove that $\dfrac{\sin 5\theta}{\sin\theta} = 16\sin^4\theta - 20\sin^2\theta + 5$.

2. Prove that $1 - \tanh^2 x = \operatorname{sech}^2 x$.

3. If $\dfrac{\tan\theta}{\theta} = \dfrac{2524}{2523}$, find θ approximately.

4. Find the equation to the plane through (2,-4,5) and is parallel to the plane $4x+2y-7z+6=0$.

5. Find the straight line through (3,2,-8) and perpendicular to $-3x+y+2z-2=0$.

6. Find the centre and radius of the sphere

$$7x^2+7y^2+7z^2+28x-42y+56z+3=0.$$

7. Find unit vector normal to the surface

$x^2+2y^2+z^2=7$ at (1,-1,2)

8. Prove that curl $(grad\phi)=\overline{0}$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

9. Prove that

$$2^5\cos^6\theta=\cos 6\theta+6\cos 4\theta+15\cos 2\theta+10.$$

10. If $\cos(x+iy)=\cos\alpha+i\sin\alpha$ prove that $\cos 2x+\cosh 2y=2$.

11. Sum the series $\sin\theta+\frac{\sin 2\theta}{2}+\frac{\sin 3\theta}{3}+....\infty$.

12. Find the equation of the plane through (2,-3,1) and is perpendicular to the line joining the points (3,4,-1) and (2,-1,5).

13. Show that the lines $\frac{x+3}{2}=\frac{y+5}{3}=\frac{3-7}{-3}$ and $\frac{x+1}{4}=\frac{y+1}{5}=\frac{3+1}{-1}$ are coplanar and find the equation of the plane containing them.

14. Find the equation of the sphere through the circle $x^2+y^2+z^2=9$, $2x+3y+4z=5$ and the point (1,2,3).

15. Show that $\overline{F}=yz\hat{i}+3x\hat{j}+xy\hat{k}$ is irrotational. Find ϕ so that $\overline{F}=\nabla\phi$.

16. Show that Green's theorem in a plane can be deduced as a special case of Stoke's theorem.

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UG–477 **BMS–13**

B.Sc. DEGREE EXAMINATION –
JANUARY 2009.

First Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Solve : $p^2 + 2xp - 3x^2 = 0$.

2. Solve : $(D^2 + 9)\, y = e^{2x} + 2x^2$.

3. Solve : $(D^2 - 4D + 3)\, y = e^x.\cos 2x$.

4. Eliminate the arbitrary functions f and g from the equation $z = f\,(x + ay) + g\,(x - ay)$.

5. Solve : $x\,(y^2 + z)\, p - y\,(x^2 + z)\, q = z\,(x^2 - y^2)$.

6. Solve : $p^3 = qz$.

7. Find $L\,[t \cos 2t]$.

8. Find $L^{-1}\left[\log\left(\dfrac{s}{s^2+1}\right)\right]$.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. Solve : $(y^4 + 2y)\,dx + (xy^3 + 2y^4 - 4x)\,dy = 0$.

10. Solve : $x^2 \dfrac{d^2y}{dx^2} - 3x \dfrac{dy}{dx} - 5y = \sin(\log x)$.

11. Solve the equation

$$(x-1)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = (x-1)^2$$

by the method of variation of parameters.

12. Solve :

$$(1 + x + x^2)\frac{d^3y}{dx^3} + (3 + 6x)\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 0.$$

13. Solve : $2xz - px^2 - 2qxy + pq = 0$.

14. Solve : $z^4q^2 - z^2p = 1$.

15. (a) Evaluate $\int_0^\infty \frac{e^{-t} \sin t}{t}\, dt$ using Laplace transforms. (5)

(b) Find $L^{-1}\left[\frac{2s+1}{(s+2)^2\ (s-1)^2}\right]$. (5)

16. Using Laplace transforms, solve the equation

$y^{(2)} + ty' - y = 0$ when $y(0) = 0,\ y'(0) = 1$.

UG–478	**BMS–21**

B.Sc. DEGREE EXAMINATION – JANUARY 2009.

Second Year

Mathematics

GROUPS AND RINGS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Show that $f : R \to R$ defined by $f(x) = 2x - 3$ is a bijection and find its inverse. Compute $f^{-1} \circ f$ and $f \circ f^{-1}$.

2. State and prove second principle of induction.

3. If A_n is the set of all even permutations in S_n, then prove that A_n is a group containing $\frac{n!}{2}$ permutations.

4. Prove that a subgroup of a cyclic group is cyclic.

5. If the index of a subgroup H of a group G is two, then show that $aH = Ha$, for every $a \in G$.

6. If G is any group and $a \in G$, then show that $\phi_a : G \to G$ defined by $\phi_a(x) = axa^{-1}$ is an automorphism of G.

7. Prove that any finite integral domain is a field.

8. Prove that any Euclidean domain R has an identity element.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. If $f : A \to B$ and $g : B \to C$ are bijections, then show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

10. If A and B are two subgroups of a group G, then prove that AB is a subgroup of G if and only if $AB = BA$.

11. If G is a group and $a, b \in G$, then show that

(a) order of a = order of a^{-1}

(b) order of a = order of $b^{-1}ab$

(c) order of ab = order of ba.

12. State and prove Lagrange's theorem. Discuss about its converse.

13. If G is a cyclic group generated by a and $f: G \rightarrow G$ is a mapping such that $f(xy) = f(x) f(y)$, then prove that f is an automorphism of G if and only if $f(a)$ is a generator of G.

14. If R is a commutative ring with identity, then prove that every maximal ideal of R is prime ideal of R.

15. Prove that for any prime p, Z_p is not an ordered integral domain.

16. Show that the ring of Gaussian integers is an Euclidean domain.

UG–479 **BMS–22**

B.Sc. DEGREE EXAMINATION – JANUARY 2009.

Second Year

Mathematics

STATISTICS AND MECHANICS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions from the following.

1. Calculate mean deviation from the median :

Class interval :	20-25	25-30	30-40	40-45	45-50
Frequency :	6	12	17	30	10
Class interval :	50-55	55-60	60-70	70-80	
Frequency :	10	8	5	2	

2. Fit a straight line trend by the method of least squares

Year	1996	1997	1998	1999	2000
Sales ('000)	4	6	7	8	10

3. The regression line of Y on X and X on Y are given by

$Y = 11.64 - 0.5\,X$ and $X = 19.13 - 0.87\,Y$

Find mean of X and Y and r.

4. Find whether A and B are independent in the following case :

$(AB) = 256, (\alpha B) = 768, (A\beta) = 48, (\alpha\beta) = 144$.

5. The following are the group index numbers and the group weights of an average working class family's budget. Construct the cost of living index number.

Group :	Food	Fuel	Clothing	Rent	Others
Index no.	352	220	230	160	190
Weight :	48	10	8	12	15

6. Suppose x is a random variable with probability mass function

$$P(x) = \frac{x^2 + 1}{148}, x = 0, 12, \cdots, 7$$
$$= 0 \qquad \textit{for } x > 7.$$

Compute $E(2x^2 + 3x)$.

7. A random sample of 600 bulbs was drawn from a large consignment and 74 was found to be defective. Find the limits of percentage of defective bulbs in the consignment.

8. Find the angle of projection when the range on a horizontal plane is $4\sqrt{3}$ times the greatest height attained.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions from the following.

9. Calculate Karl Pearson's coefficient of skewness for the following data :

Years :	0-5	5-10	10-15	15-20	20-25	25-30	30-35
Frequency :	449	705	507	281	109	53	11

10. Find the Karl Pearson's coefficient of correlation between the variables X and Y.

X:	71	68	66	67	70	71	70	73	72	65	66
Y:	69	64	65	63	65	62	65	64	66	59	62

11. Find the function U_x from the data below and hence estimate the value for $x = 2$.

x:	0	1	4	5
U_x:	8	11	78	123

12. Compute (a) Laspeyre's (b) Paasche's and (c) Fisher's index numbers

Item	Price		Quantity	
	Base	Current	Base	Current
A	6	10	50	50
B	2	2	100	120
C	4	6	60	60
D	10	12	30	25

13. Fit a Poisson distribution to the following data :

x:	0	1	2	3	4	5
f:	142	156	69	27	5	1

14. The following data related to Production in kg of three varieties A, B and C of paddy sown in 12 plots.

A	14	16	18	–	–
B	14	13	15	22	–
C	18	16	19	19	20

Is there any significance in the production of the three varieties?

15. Two smooth spheres of masses m_1 and m_2 and coefficient of restitution e, collide obliquely with velocities u_1, u_2 whose directions are inclined to the common normal at angles α_1, α_2. Find the velocities of the spheres after impact. Also find the impulse of the blow on the sphere of mass m_1.

16. Obtain the differential equation of a central orbit in $p - r$ coordinates.

UG–474	**BMSA-01**

B.Sc. DEGREE EXAMINATION
JANUARY 2009.

(AY – 2005-06 and CY – 2006 batches only)

Third Year

Mathematics

GRAPH THEORY

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Show that the sum of degrees of the points of a graph G is twice the number of lines.

2. Show that every (p, q)-graph with $q \geq p$ contains a cycle.

3. Write down the Fleury's algorithm.

4. Let G be a graph with p points and let u and v be a nonadjacent points in G such that $d(u)+d(v) \geq p$. Show that G is Hamiltonian if and only if $G+uv$ is Hamiltonian.

5. Prove that every connected graph has a spanning tree.

6. Show that any tree S constructed by Prim's algorithm is an optimal tree.

7. Prove that every planner graph G with $p \geq 3$ points has at least three points of degree less than 6.

8. Show that there exists a k-colouring of a graph G *if and only if* $V(G)$ *can be partitioned into* k *subsets* $V_1, V_2 ...., V_k$ *such that no two vertices in* V_i, $i = 1, 2, ...k$ are adjacent.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. (a) Define :

(i) Null graph,

(ii) Sub graph and spanning sub graph.

(b) In any graph G, show that the number of points of odd degree is even.

10. Prove that every non trivial graph contains at least two vertices which are not cut vertices.

11. Prove that a connected graph is Eulerian if and only if it has no vertex of odd degree.

12. A (p,q)-graph G is a bipartite graph if and only if it contains no odd cycles.

13. State and prove the Hall's theorem.

14. For any graph G prove that $\psi(G) \leq \Delta(G)+1$.

15. Show that the digraph D is strongly connected if and only if D contains a directed closed walk containing all its vertices.

16. Show that every strong tournament D on $p \geq 3$ vertices contains a directed cycle of length k, for every k, $3 \leq k \leq p$.