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| **PG–687** | **MMS–11** |

M.Sc. DEGREE EXAMINATION –
JANUARY 2009.

First Year

(AY 2005-06 and CY 2006 batches only)

Mathematics

ALGEBRA

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Prove that centre of a group $G$ is a normal subgroup of $G$.

2. Prove that conjugacy relation is an equivalence relation on a group $G$.

3. Prove that a group of order $p^n$ is nilpotent where $p$ is prime.

4. Let $f$ be a homomorphism of a ring $R$ into $R'$. Show that $f$ is one-one iff ker.$f$. = 0.

5. Show that every Euclidean ring possesses a unit element.

6. Prove that the polynomial

   $f(x) = (1 + x + x^2 + ....... + x^{p-1}) \in Q[x]$

   is irreducible.

7. Let $f(x) \in F[x]$ be of degree $n$. Show that $f(x)$ has a splitting field.

8. Show that every finite extension of a finite field is Galois.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. If $H$ and $K$ are normal subgroups of $G$, show that both $HK$ and $H \cap K$ are normal subgroups of $G$.

10. If $G$ is a finite group and $p$ is prime, show that $G$ has a subgroup of order $p^{\sim}$ where $p^{\sim} \mid 0(G)$.

11. Prove that every finite integral domain $R$ is a field. Also show that $Z_p$ is a field.

12. State and prove unique factorization theorem on Euclidean rings.

13. Prove that product of two primitive polynomials over unique factorization domain is also a primitive polynomial.

14. Let $K/F$ be a finite separable extension. Show that $K = F(\sim)$ for some $\sim \in K$.

15. Let $F \subseteq L \subseteq K$ where $K/F$ is Galois extension. Show that $L/F$ is normal extension iff $G(K/L)$ is a normal subgroup of $G(K/F)$.

16. Let $K/F$ be a Galois radical extension. Show that $G(K/F)$ is a solvable group.

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**PG–688** **MMS–12**

M.Sc. DEGREE EXAMINATION –
JANUARY 2009.

First Year

(AY 2005–06 and CY 2006 batches only)

Mathematics

REAL AND COMPLEX ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

17. If $X$ is a compact metric space, and $\{p_n\}$ is a Cauchy sequence in $X$, prove that $\{p_n\}$ converges to some point of $X$.

18. If the partial sums $A_n$ of $\sum a_n$ form a bounded sequence and $b_0 \geq b_1 \geq b_2 \geq .....$ and $\lim_{n \to \infty} b_n = 0$, prove that $\sum a_n b_n$ converges.

19. Let $f$ be a continuous real function on the interval $[a, b]$. If $c$ is a number satisfying $f(a) < c < f(b)$, show that there exists a point $x \in (a, b)$ such that $f(x) = c$.

20. State and prove the fundamental theorem of calculus.

21. Show that $u(x, y) = x^3 - 3xy^2$ is harmonic and find the analytic function $f(x) = u + iv$.

22. Prove that every totally bounded set is bounded.

23. Define $n(¥, a)$ and prove that it is an integer.

24. State and prove Hurwitz theorem for sequence of analytic functions.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

25. (a) Let $\{s_n\}$ be monotonic. Prove that $\{s_n\}$ converges if and only if it is bounded.

(b) Prove that the product of two convergent series is convergent if atleast one of the two series converges absolutely.

26. Let $f$ be monotonically increasing on $(a, b)$. Show that $f(x+)$ and $f(x-)$ exist at every point of $x$ of $(a, b)$.

27. State and prove Taylor's theorem for real functions.

28. Show that $f \in R(\sim)$ on $[a, b]$ if and only if for $\in > 0$, there exists a partition $p$ satisfying $U(P, f, \sim) - L(P, f, \sim) < £$.

29. For every power series $\sum_{n=0}^{\infty} a_n z^n$, there exists a number $R$, $0 \le R \le \infty$, prove the following :

(a) The series converges absolutely for every $z$ with $|z| < R$.

(b) If $|z| < R$, the sum of the series $\sum a_n z^n$ is an analytic function.

30. Prove that the cross ratio $(z_1, z_2, z_3, z_4)$ is real if and only if the four points lie on a circle or on a straight line.

31. (a) State and prove Liouville's theorem. Deduce the fundamental theorem of algebra.

(b) If $f(z)$ is a non-constant analytic function in a region $\Omega$, prove that its absolute value $|f(z)|$ has no maximum in $\Omega$.

32. Establish Laurent's series for an analytic function in an annulus.

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M.Sc. DEGREE EXAMINATION –
JANUARY 2009.

First Year

(AY 2005–2006 and CY 2006 batches only)

Mathematics

TOPOLOGY AND MEASURE THEORY

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

33. Let $\{A_\alpha\}$ be a collection of subsets of a space $X$. Then prove that $\overline{\cup A_\alpha} \supset \cup \overline{A_\alpha}$ and give an example where the equality fails.

34. Let $X$ be a metric space with metric $d$. Define $\overline{d} : X \times X \to \mathbf{R}$ by the equation :

$\overline{d}(x, y) = \min\{d(x, y), 1\}$ .

Then prove that $\overline{d}$ is a metric that induces the topology of $X$.

35. Let $X$ be a non-empty compact Hausdorff space. If every point of $X$ is a limit point of $X$, then prove that $X$ is uncountable.

36. Prove that subspace of a regular space is regular and a product of regular spaces is regular.

37. Prove that the Lebesgue outer measure $m^*$ is translation invariant.

38. Show that if $f$ is integrable over $E$, then so is $|f|$ and $\left|\int_E f\right| \leq \int_E |f|$.

39. State and prove the Fatou's Lemma.

40. Define :

(a) Signed measure

(b) Positive set

(c) Null set

(d) Negative set

(e) Total variation of a signed measure.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

41. (a) Define square metric ° in $\mathbf{R}^n$.

(b) Prove that the topologies on $\mathbf{R}^n$ induced by the Euclidean metric $d$ and the square metric ° are the same as the product topology on $\mathbf{R}^n$.

42. State and prove the Lebesgue number lemma.

43. (a) Define linear continuum.

(b) If $L$ is a linear continuum in the order topology, then prove that $L$ is connected. Also prove every interval and ray in $L$ are connected.

44. (a) Prove that every metrizable space is normal. (3)

(b) Prove that every compact Hausdorff space is normal. (3)

(c) Prove that every regular space with a countable basis is normal. (4)

45. (a) Prove that the outer measure of an interval is its length. (7)

(b) Prove that the Lebesgue outer measure $m^*$ is countable subadditive. (3)

46. (a) State and prove the bounded convergence theorem. (5)

(b) State and prove the Fatous Lemma. (3)

(c) State and prove the monotone convergence theorem. (All with respect to Lebesgue measure). (2)

47. State and prove the Radon-Nikodym theorem.

48. Prove that the class $\mathbf{B}$ of «$^*$–measurable sets is a ± algebra. If « is restricted to $\mathbf{B}$, then show that « is a complete measure on $\mathbf{B}$.

# M.Sc. DEGREE EXAMINATION – JANUARY 2009.

First Year

(AY 2005–2006 and CY 2006 batches only)

Mathematics

## NUMERICAL METHODS AND DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

49. Using Secant method, find the roots of $x^4 - x - 10 = 0$ correct to three decimal places.

50. By Gauss-elimination method, solve the equations:

$$\begin{aligned} 4x_1 + x_2 + x_3 &= 4, \\ x_1 + 4x_2 - 2x_3 &= 4, \\ 3x_1 + 2x_2 - 4x_3 &= 6. \end{aligned}$$

51. Find the unique polynomial $P(x)$ of degree 2 or less such that $P(1) = 1$, $P(3) = 27$, $P(4) = 64$, using Newton-divided difference method.

52. Explain comosite Simpson's rule to find $\int_a^b f(x)dx$.

53. State and prove uniqueness theorem on the solution of $n$th order differential equation.

54. Find all solutions of the equation :

$x^3 y''' + 2x^2 y'' - xy' + y = 0$ for $x > 0$.

55. Suppose $S$ is a rectangle $|x - x_0| \le a$, $|y - y_0| \le b$, $(a, b > 0)$ and that $f(x, y)$ is a real valued function defined on $S$ such that $\frac{\partial f}{\partial y}$ exists, is continuous on $S$ and $\left|\frac{\partial f}{\partial y}\right| \le K \; ((x, y) \text{ in } S)$, $K > 0$. Prove that $f$ satisfies Lipschitz condition on $S$.

56. Find the general solution of the equation

$(y + zx)p - (x + yz)q = x^2 - y^2$.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

57. Using Chebyshev method, find the root of the equation $\cos x - xe^x = 0$ correct to four decimal places.

58. Use the Gauss-Seidel method to find the solution of the system of equations :

$$\begin{aligned} 2x_1 - x_2 &= 7, \\ -x_1 + 2x_2 - x_3 &= 1, \\ -x_2 + 2x_3 &= 1. \end{aligned}$$

59. Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ using mid-point method with $h$ = 0.2 over the interval [0, 1].

60. Evaluate the integral $\int_0^1 \frac{dx}{1+x}$ using composite trapezoidal rule.

61. If $¤_1, ¤_2, .......¤_n$ are $n$ solutions of $L(y) = 0$ on an interval I, prove that they are linearly independent there if and only if $W(¤_1, ......., ¤_n)(x) \neq 0$ for all $x \in I$

62. Obtain the Bessel function of zero order of the first kind $J_0(x)$.

63. Compute the first four successive approximation $¤_0, ¤_1, ¤_2, ¤_3$ for the problem $y' = x^2 + y^2,\ y(0) = 0$.

64. Show that $xp - yq = x,\ x^2 p + q = xz$ are compatible and find their solution.

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**PG–696 MMS–16**

M.Sc. DEGREE EXAMINATION –
JANUARY 2009.

First Year

(AY 2006–07 batch onwards)

Mathematics

## REAL ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

65. Show that compact subset of metric spaces are closed.

66. If $X$ is a compact metric space and if $\{p_n\}$ is a Cauchy sequence in $X$, show that $\{p_n\}$ converges to some point of $X$.

67. Show that continuous image of a compact space is compact.

68. Let $f$ be defined on $[a,b]$ such that $f$ has a local maximum at a point $x \in (a,b)$ and $f'(x)$ exists. Show that $f'(x) = 0$.

69. Show that every continuous function defined on $[a, b]$ is Riemann Steltjes integrable.

70. Suppose $K$ is compact and $\{f_n\}$ is a sequence of continuous functions on $K$, $\{f_n\}$ converges pointwise to a continuous function on $K$ and $f_n(x) \geq f_{n+1}(x)$ for all $x \in K$. Show that $f_n \to f$ uniformly on $K$.

71. Let $A$ be an algebra of real continuous functions on a compact set $K$ and $A$ separates points on $K$ and $A$ vanishes at no point of $K$. If $B$ is the uniform closure of $A$ show that for a real function $f$ continuous on $K$ and $x \in K$, $\in > 0$ show that there exists a function $g_x \in B$ such that $g_x(x) = f(x)$ and $g_x(t) > f(t) - \in$ for $t \in E$.

72. Suppose $f_n$ defined in an open set $E \subset R^2$, suppose that $D_1 f, D_{21} f$ and $D_2 f$ exist at every point of $E$ and $D_{21} f$ is continuous at some point $(a,b) \in E$. Show that $D_{12} f$ exists at $(a,b)$ and $\left(D_{12} f\right)(a,b) = (D_{21} f)(a,b)$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

73. Define connected subset of a metric space. Show that a subset $E$ of the real line $R'$ is connected if and only if it has the following property : if $x, y \in E$ and $x < z < y$, then $z \in E$.

74. (a) Suppose $a_1 \geq a_2 \geq ....... \geq 0$. Show that the series $\sum_{n=1}^{\infty} a_n$ converges if and only if the series $\sum_{k=0}^{\infty} 2^k a_{2^k}$ converges.

(b) State and prove Ratio Test.

75. (a) Show that continuous image of a connected space is connected.

(b) Let $f$ be monotonic on $(a, b)$. Show that the set of all points of $(a, b)$ at which $f$ is discontinuous is at most countable.

76. State and prove Taylor's theorem.

77. (a) Let $f \in R$ on $[a, b]$ and $F(x) = \int_a^x f(t)\, dt$ for $a \leq x \leq b$. Show that $F$ is continuous on $[a, b]$ and if $f$ is continuous at a point $x_0 \in [a, b]$ show that $F$ is differentiable at $x_0$ an $F'(x_0) = f(x_0)$.

(b) State and prove Fundamental theorem of calculus.

78. Show that there exists a real continuous function on the real line which is nowhere differentiable.

79. Suppose $f$ maps an open set $E \subset R^n$ into $R^m$. Show that $f \in \P^1(E)$ if and only if the partial derivatives $D_j f_i$ exist and are continuous on $E$ for $1 \leq i \leq m$, $1 \leq j \leq n$.

80. State and prove Rank theorem.

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| **PG–697** | **MMS–17** |

M.Sc. DEGREE EXAMINATION –
JANUARY 2009.

First Year

(AY 2006–07 batch onwards)

Mathematics

# COMPLEX ANALYSIS AND NUMERICAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

81. Explain stereographic projection.

82. Let $f(z) = \left(\frac{\bar{z}}{z}\right)^2$, $z \neq 0, f(0) = 0$. Show that C-R equations are satisfied at (0, 0) but $f$ is not differentiable at the origin.

83. State and prove Schwarz lemma.

84. Find the Taylor series to represent $\frac{z^2 - 1}{(z+2)(z+3)}$ in $|z| < 2$.

85. Solve by Gauss-Jordan method

$$\begin{aligned} 10x_1 + x_2 + x_3 &= 12 \\ x_1 + 10x_2 + x_3 &= 12 \\ x_1 + x_2 + 10x_3 &= 12. \end{aligned}$$

86. Given :

| | | | | |
|---|---|---|---|---|
| $x$ : | 5 | 6 | 9 | 11 |
| $f(x)$ : | 12 | 13 | 14 | 16 |

Find the value of $y$ when $x = 10$, using Lagrange's formula for unequal intervals.

87. Apply Simson's three-eight rule to evaluate the approximate value of $\int_0^6 \frac{dx}{1+x^2}$ by dividing the range into six equal parts.

88. Explain Picard's method of successive approximations.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

89. State and prove a set of sufficient conditions for $f(z)$ to be analytic in a region.

90. Define a conformal mapping. If $f(z)$ is analytic at $z$ and $f'(z) \neq 0$ in a region $D$, prove that $f(z)$ is conformal at all points in $D$.

91. State and prove the maximum principle for an analytic function on a closed bounded set $E$.

92. By method of residues, show that $\int_0^\infty \frac{x^{p-1}}{1+x} = \frac{®}{\sin p®}$, $0 < p < 1$.

93. Solve the following equation using Gauss-Jacobi iteration method.

$$
\begin{aligned}
20x + y - 2z &= 17 \\
3x + 20y - z &= -18 \\
2x - 3y + 20z &= 25.
\end{aligned}
$$

94. Using Gauss-Seidel method, solve the system of equations :

$$
\begin{aligned}
8x - y + z &= 18 \\
2x + 5y - 2z &= 3 \\
x + y - 3z &= -6.
\end{aligned}
$$

95. Given $u_{75} = 2459$, $u_{80} = 2018$, $u_{85} = 1180$ and $u_{90} = 402$, Find $u_{79}$ by Newton's formula.

96. Solve $y' = x\,y$, $y(1) = 2$, for $x = 1.4$ using Runge-Kutta methods.

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| **PG–692** | **MMS–22** |

M.Sc. DEGREE EXAMINATION –
JANUARY 2009.

Second Year

(AY 2005–06 & CY–2006 batches only)

Mathematics

OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

97. Define :

(a) Slack variable

(b) Surplus variable

(c) Artificial variable.

98. Explain the dual simplex method.

99. Write Dijkstra's algorithm.

100. Discuss dynamic programming with suitable examples.

101. Explain the two-person zero-sum game giving a suitable example.

102. What do you understand by a queue? Give some important applications of queuing theory.

103. Explain what is meant by Kuhn-Tucker conditions.

104. Explain : Markov process.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

105. Use penalty ( or Big M ) method to Maximize

$z = 3x_1 - x_2$

Subject to the constraints

$$2x_1 + x_2 \geq 2$$
$$x_1 + 3x_2 \leq 3$$
$$x_2 \leq 4$$

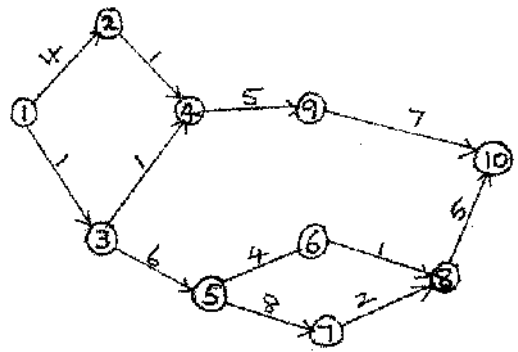
where $x_1, x_2 \geq 0$.

106. Carry out three iteration of Karmarkar's algorithm for the following problem.

Maximize $z = x_1 - 2x_2$

Subject to
$$x_1 - 2x_2 + x_3 = 0$$
$$x_1 + x_2 + x_3 = 1$$
where $x_{1,} x_2, x_3 \geq 0$

107. Determine the critical path for the project network in figure.



108. Use dynamic programming to solve the following problem.

Minimize $z = y_1^2 + y_2^2 + y_3^2$

Subject to the constraints

$y_1 + y_2 + y_3 \geq 15$

and $y_1, y_2, y_3 \geq 0$.

109. Solve the game whose pay-off matrix is given by

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 2 & 3 & 1/2 \\ 3/2 & 2 & 0 \\ 1/2 & 1 & 1 \end{bmatrix} \end{array}$$

110. On an average, 6 customers reach a telephone booth every hour to make calls. Determine the probability that exactly 4 customers will reach in 30-minute period, assuming that arrivals follow Poisson distribution.

111. Solve the following using Jacobian method.

Minimum $f(x) = x_1^2 + x_2^2 + x_3^2$

Subject to $g_1(x) = x_1 + x_2 + 3x_3 - 2 = 0$

$g_2(x) = 5x_1 + 2x_2 + x_3 - 5 = 0.$

112. Consider a Markov chain with two states.

$$P = \begin{bmatrix} 0.1 & 0.9 \\ 0.7 & 0.3 \end{bmatrix}$$

with $a^{(0)} = (0.8, 0.2)$. Determine $a^{(1)}, a^{(4)}$ and $a^{(8)}$.

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| **PG–694** | **MMS–24** |

M.Sc. DEGREE EXAMINATION –
JANUARY 2009.

Second Year

(AY 2005–06 & CY– 2006 batches only)

Mathematics

FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

113. If $M$ is a closed linear subspace of a normed linear space $N$ and if $T$ is the natural mapping of $N$ onto $N/M$ defined by $T(x)=x+M$. Show that $T$ is a continuous linear transformation for which $\|T\|\leq 1$.

114. If $P$ is a projection on a Banach space $B$ and if $M$ and $N$ are its range and null spaces, show that $M$ and $N$ are closed linear subspaces of $B$ such that $B=M\oplus N$.

115. Show that in a separable Hilbert space $H$, every orthonormal set in $H$ is countable.

116. If $\{e_i\}$ is an orthonormal set in a Hilbert space, show that the set $\{e_i : (x, e_i) \neq 0\}$ for $x \in H$ is either empty or countable.

117. Let $T \in B(H)$. Show that $T=0$ if and only if $(T_x, x)=0$ for all $x \in H$.

118. If $T$ is an arbitrary operator on $H$ and if $\sim$,   are scalars such that $|\sim| = |\ \ |$, show that $\sim T + \ \ T^*$ is normal.

119. Show that the non zero characteristic vectors $x_1, x_2, \ldots\ldots, x_n$ corresponding to characteristic values $a_1, a_2, \ldots\ldots, a_n$ of a linear operator $T$ on $H$ are linearly independent.

120. Show that the boundary of the set of all singular elements is a subset of the set of all topological divisor of zero.

SECTION B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

121. Let $N$ be a normed linear space and $N'$ be a Banach space. Show that $B(N,N')$ is also Banach space.

122. State and prove open mapping theorem.

123. Let $M$ be a closed subspace of a Hilbert space $H$. Let $x \notin M$. Let $d$ be the distance between $x$ and $M$. Show that there exists a unique element $y_0 \in M$, such that $\|x - y_0\| = d$.

124. Explain the Gram Schmitch orthogonalization process of constructing an ortho normal set from a given linearly independent set of vectors in a Hilbert space.

125. If $P$ is a projection on $H$ with range $M$ and null space $N$, show that $M \perp N$ if and only if $P$ is self adjoint and $N = M^{\perp}$.

126. (a) If $P$ is a projection on a closed linear subspace $M$ of a Hilbert space $H$, show that $M$ reduces an operator T if and only if $TP = PT$.

(b) If $P$ and $Q$ are perpendicular projections on closed linear subspaces $M$ and $N$ respectively of a Hilbert space $H$, show that $M$ and $N$ are orthogonal if and only if $PQ = 0$ if and only if $QP = 0$. (5 + 5)

127. (a) If $f_1$ and $f_2$ are multiplicative functionals on a commutative Banach algebra $A$ with the same null space $n$, show that $f_1 = f_2$.

(b) If $A$ is a semi simple commutative Banach Algebra, show that the involution on $A$ is continuous. (5 + 5 )

128. State and prove Gelfand Neumark theorem.

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**PG–700 MMS–26**

M.Sc. DEGREE EXAMINATION –
JANUARY 2009.

Second Year

(AY 2006–07 batch onwards)

Mathematics

OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

129. Obtain the dual of the following linear programming problem :

Maximize $Z = x_1 - x_2 + 3x_3$

Subject to the constraints :

$$x_1 + x_2 + x_3 \leq 10$$
$$2x_1 - x_3 \leq 2$$
$$2x_1 - 2x_2 + 3x_3 \leq 6$$

and $x_1, x_2, x_3 \geq 0$.

130. Explain the basic idea of the interior point used in Karmarkar algorithm.

131. Explain the basic idea of the Acyclic algorithm.

132. Use arithmetic method to solve the following game: $\begin{pmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}$.

133. Find the range of values of $p$ and $q$ which will render the entry (2, 2) a saddle point for the game.

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} 2 & 4 & 5 \\ 10 & 7 & q \\ 4 & p & 5 \end{pmatrix} \end{array}$$

134. Explain the pure birth and pure death processes.

135. Define Hessian matrix. Write the sufficient condition for a stationary point $x_0$, to be an extremum.

136. Explain : Geometric programming.

SECTION B — ($5 \times 10 = 50$ marks)

Answer any FIVE of the following.

137. Use Simplex method to solve the following linear programming problem.

Maximize $Z = x_1 + 4x_2 + 5x_3$

Subject to the constraints :

$$3x_1 + 3x_3 \leq 22$$
$$x_1 + 2x_2 + 3x_3 \leq 14$$
$$3x_1 + 2x_2 \leq 14$$
and $x_1, x_2, x_3 \geq 0$.

138. Use two phase Simplex method to solve the following linear programming problem :

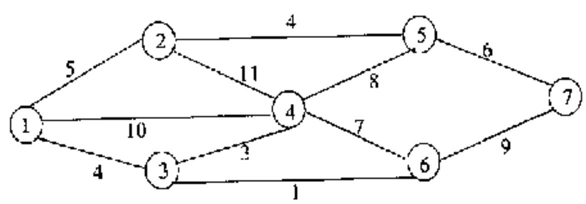
Minimize $Z = x_1 + x_2$

Subject to the constraints :

$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$

and $x_1, x_2 \geq 0$.

139. Determine the shortest-route of the network using acyclic algorithm.




140. Use Dynamic programming techniques to solve the following :

Maximize $Z = x_1^2 + 2x_2^2 + 4x_3$

Subject to the constraints :

$x_1 + 2x_2 + x_3 \leq 8$ and $x_1, x_2, x_3 \geq 0$.

141. Solve the game whose pay-off matrix is

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} 1 & 4 & -2 & -3 \\ 2 & 1 & 4 & 5 \end{pmatrix} \end{array}$$

142. Customers at a petrol bunk with one servicing counter arrive at a rate of 20 per hour and the average number of customers that can be processed is 24 per hour. Find :

(a) The probability that the server is idle.

(b) The average number of customers in the queue, and

(c) The average time the customer has to spend in the queue.

143. Determine $x_1$ and $x_2$ so as to maximize $Z = 12x_1 + 21x_2 + 2x_1x_2 - 2x_2^2$ subject to the constraints: $x_2 \le 8$, $x_1 + x_2 \le 10$ and $x_1, x_2 \ge 0$.

144. Using Steepest ascent method, solve the following non-linear programming problem.

Maximise $f(x_1, x_2) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$.

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**PG–702 MMS–28**

M.Sc. DEGREE EXAMINATION –
JANUARY 2009.

Second Year

(AY 2006–07 batch onwards)

Mathematics

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

145. Define the Wronskian of $n$ functions defined on an interval $I$. Verify whether the functions $e^x$, $e^{-x}$ are linearly independent or not.

146. Solve : $y'' - 2y' - 3y = 0$, $y(0) = 0$, $y'(0) = 4$.

147. If $P_m(x)$ and $P_n(x)$ are Legendre polynomials, prove that $\int_{-1}^{1} P_n(x)\,P_m(x)\,dx = 0$, if $m \ne n$.

148. Compute the indicial polynomial and the roots for the equation :

$$4x^2y'' + (4x^4 - 5x)y' + (x^2 + 2)y = 0$$

149. Let $A(x)$ be a continuous $n \times n$ matrix such that $A(x + w) = A(x)$, $w \ne 0$, $-\infty < x < \infty$. Let $\Phi(x)$ be a fundamental matrix of $y' = Ay$. Prove that $\Phi(x + w)$ is also a fundamental matrix of the system of equations.

150. Define Lipschitz condition for $f(x, y)$. Show that the function $f(x, y) = y^{1/2}$ does not satisfy the Lipschitz condition on $R : | x | \leq 1,\ 0 \leq y \leq 1$.

151. Find a particular integral of the equation $\left(D^2 - D'\right)z = e^{x+y}$.

152. Show that the one-parameter family of surfaces $x^2 + y^2 = c\,z^2$ can form a family of equipotential surfaces.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

153. Let ¤ be any solution of the equation $L(y) = y^{(n)} + a_1\, y^{(n-1)} + \dots\dots + a_n y = 0$ on an interval I containing a point $x_0$. Prove that, for all $x$ in $I$,

$$\| ¤(x_0) \|\, e^{-k\{x-x_0|} \leq \| ¤\,(x) \| \leq \| ¤\,(x_0) \|\, e^{k|x-x_0|}$$

Where $k = 1 + | a_1 | + | a_2 | + \dots\dots + | a_n |$.

154. Explain the variation of parameter method to find the general solution of the equation, $y'' + a_1(x)y' + a_2(x)y = b(x)$.

155. Obtain the power series solution of the equation :

$(1 - x^2)y'' - 2xy' + \sim(\sim + 1)y = 0$, where $\sim$ is a constant.

156. Derive the Bessel function of order $\sim$ of the first kind $J_{\sim}(x)$.

157. Let $A$ be a constant matrix. Derive the solution of the system $y' = Ay$ on $I$, with the initial condition $y(x_0) = y_0$, $x, x_0 \in I$.

158. State and prove Picard's theorem on existence and uniqueness of solution of initial value problems.

159. Using Riemann's method, discuss the solution of hyperbolic equation :

$$\frac{\partial^2 z}{\partial x\, \partial y} + a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} + cz = f(x, y).$$

160. State and prove Kelvin's inversion theorem.