PG–237	**MMS–11**

M.Sc. DEGREE EXAMINATION – JUNE 2009.

(AY 2005-2006 and CY 2006 batches only)

First Year

Mathematics

ALGEBRA

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Show that N of a group G is a normal subgroup of G if and only if every left coset of N in G in a right coset of N in G.

2. Show that every permutation is the product of its disjoint cycles.

3. Let R be a commutative ring with unit element whose only ideals are *(o)* and R itself. Show that R is field.

4. Let R be an Euclidean ring and $a, b \in R$. If $b \neq o$ is not a unit in R show that $d\ (a) < d\ (a,b)$.

5. Let V be a vector space over a field F and let $S, T \subset V$. Show that $L\ (SUT) = L(S) + L\ (T)$.

6. Show that $\sqrt{2} + \sqrt[3]{5}$ is algebraic over Q. Find its degree.

7. Find the splitting field of the polynomial $x^4 + 1$ over Q.

8. If K is a field of complex numbers and F is a field of real number compute $G\ (K : F)$.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. State and prove fundamental theorem of group homomorphism.

10. If p is a prime number and $p^{\sim} / o(G)$ show that G has a subgroup of order $p^{\sim}$.

11. Show that every integral domain can be imbedded in a field.

12. If F is a field show that $F(x)$ is an Euclidean ring.

13. (a) If V is a finite dimensional vector space and W is a subspace of V, show that $\dim\left(\frac{V}{W}\right) = \dim V - \dim W$.

(b) If V is finite dimensional show that V is isomorphic to $\hat{\hat{V}}$.

14. Show that $a \in K$, a field is algebraic over F if and only if $F(a)$ is finite extension of F.

15. If $p(x)$ is irreducible in $F(x)$ and if v is a root of $p(x)$ show that $F(v)$ is isomorphic to $F'(w)$ where w is a root of $p'(+)$ and the isomorphism $\pm : F(^3) \to F'(w)$ be so choosen such that $\pm(^3) = w$ and $\pm(\sim) = \sim'$ for $\sim \in F$.

16. If K is a normal extension of F and H is a subgroup of $G(K : F)$. Let $K_H = \{x \in H : \pm(x) = x \; \forall \pm \in K\}$. Show that

(a) $[K : K_H] = o(H)$

(b) $H = G(K : K_H)$.

PG–238	**MMS-12**

M.Sc. DEGREE EXAMINATION – JUNE 2009.

(AY 2005-06 and CY 2006 batches only)

First Year

Mathematics

REAL AND COMPLEX ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

17. If $x, y \in R$; $x < y$, then prove that there exists a $p \in Q$ such that $x < p < y$.

18. State and prove the root test for convergence of the series $\sum a_n$.

19. Let f be a continuous real function on the interval $[a,b]$. If $f(a) < f(b)$, and c is a number satisfying $f(a) < c < f(b)$ show that there exist a point $x \in (a,b)$ such that $f(x) = c$.

20. If p^* is a refinement of p, then prove that $L(p, f, \sim) \leq L(p^*, f, \sim)$.

21. Find the values of $(-i)^{1/4}$.

22. Show that $f(z) = z^2$ is an analytic function.

23. Prove that every totally bounded set is bounded.

24. State and prove Schwartz's lemma.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

25. In any metric space X, prove that every convergent sequence is a Cauchy sequence. If X is a compact metric space, prove that every Cauchy sequence converges to some point of X.

26. Show that $\sum \frac{1}{n^p}$ converges if $p > 1$, and diverges if $p \leq 1$.

27. Show that a mapping f of a metric space X into a metric space Y is continuous on X iff $f^{-1}(V)$ is open in X for every open set V in Y.

28. State and prove the fundamental theorem of calculus.

29. If u and v have first-order partial derivatives that satisfy the Cauchy-Riemann equations, then show that $f(z) = u + iv$ is analytic.

30. If $f : X \to Y$ is continuous, and X is compact, prove that f is uniformly continuous.

31. State and prove Cauchy's theorem for a rectangle.

32. Let z_j be the zeros of a function $f(z)$ that is analytic in a disk and $\not\equiv 0$, each zero being counted as many times as its order indicates. For every closed curve r which does not pass through a zero, prove that $\sum_j n(r, z_j) = \frac{1}{2® i} \int_r \frac{f'(z)}{f(z)} dz$.

PG–239	**MMS–13**

M.Sc. DEGREE EXAMINATION – JUNE 2009.

First Year

(AY – 2005-06 and CY – 2006 batches only)

Mathematics

TOPOLOGY AND MEASURE THEORY

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

33. Let X and Y be topological spaces; let $f : X \to Y$. Then the following are equivalent :

(a) f is continuous

(b) for every subset A of X, one has $f(\overline{A}) \subseteq \overline{f(A)}$.

(c) for every closed set B in Y, the set $f^{-1}(B)$ is closed in X.

34. Prove that the union of a collection of connected sets that have a point in common is connected.

35. Prove that compact subset of a Hausdorff space is closed.

36. Suppose that X has a countable basis. Then prove that (a) every open covering of X contains a countable subcollection covering X. (b) There exists a countable subset of X which is dense in X.

37. Let $\{A_n\}$ be a countable collection of sets of real numbers. Then prove that $m^*(\cup A_n) \le \Sigma m^*(A_n)$.

38. State and prove the bounded convergence theorem.

39. If $E_i \in \mathbf{B}$, « $E_1 < \infty$ and $E_i \supset E_{i+1}$ then prove that « $\left(\bigcap_{i=1}^{\infty} E_i \right) = \lim_{n \to \infty}$ « E_n.

40. If f and g are measurable functions then prove that $f + g$ and $f \cdot g$ are measurable.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

41. Let $\overline{d}\,(a, b) = \min\{|\,a - b\,|, 1\}$ be the standard bounded metric on $\mathbf{R}$. If x and y are two points of $\mathbf{R}^{w}$, define $D(x, y) = \text{lub}\left\{ \dfrac{\overline{d}\,(x_i, y_i)}{i} \right\}$. Then prove that D is a metric that induces the product topology on $\mathbf{R}^{w}$.

42. Prove that the Cartesian product of connected spaces is connected.

43. (a) State maximum and minimum value theorem. (4)

(b) Let X be a compact Hausdorff space. If every point of X is a limit point of X then prove that X is uncountable. (6)

44. (a) Prove that every compact Hausdorff space is normal. (4)

(b) Prove that every regular space with a countable basis is normal. (6)

45. Prove that the collection M of all measurable sets form a ± – algebra.

46. Let f be defined and bounded on a measurable set E with mE finite. Prove that

$$\inf_{f \leq \cdot} \int_E \cdot\ (x)\, dx = \sup_{f \geq \text{¨}} \int_E \text{¤}\ (x)\, dx$$

for all simple functions ¨ and · , it is necessary and sufficient that f be measurable.

47. (a) State and prove the Fatou's lemma for the general measure. (6)

(b) State and prove the Lebesgue convergence theorem. (4)

48. State and prove the Radon-Nikodym theorem.

PG–240 **MMS–14**

M.Sc. DEGREE EXAMINATION – JUNE 2009.

(AY 2005–06 and CY 2006 batches only)

First Year

NUMERICAL METHODS AND DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

49. Derive iteration formula to compute $\sqrt{N}(N > 0)$, using Newton's method. Hence compute $\sqrt{6}$, corrected to 6 decimal places.

50. Solve the system of equations $x + 2y + z = 5$; $2x + 3y - z = 7$; $2x - y + 3z = 12$, Using Gauss – Jordan method.

51. Calculate the n^{th} divided difference of $\frac{1}{x}$.

52. Evaluate $\int_{-1}^{1} \left(1 - x^2\right)^{3/2} \cos x \, dx$, using Gauss–Legendre 3 pt. formula.

53. Show that the differential equation $y'' + \frac{y'}{x} - \frac{y}{x^2} = 0, (x > 0)$ has solution of the form x^r, r is a constant.

54. Find singular points of $x^2 y'' + (x + x^2)y' - y = 0$ and classify the same.

55. Solve the differential equation $y' = \left(\frac{3x^2 - 2xy}{x^2 - 2y}\right)$.

56. Find the complete integral of $p^2 q(x^2 + y^2) = p^2 + q$.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

57. By Tricurgularization method, solve the system of equations

$x + 2y + 3z = 10$

$2x - 3y + z = 1$

$3x + y - 2z = 9$.

58. Find inverse of the matrix

$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$ by the partition method.

59. Find approximate value of the integral $\int_0^1 \frac{dx}{1+x}$ using composite Simpson's rule with 3,5 and
9 nodes and Romberg integration.

60. Solve the initial value problem

$u' = -2tu^2, u(0) = 1$ with $h = 0.2$ on the interval [0, 0.4], using second order implicit Runge–Kutta method.

61. Find two linearly independent power series solutions of the equation

$y'' - xy' + y = 0$.

62. Prove that Bessel function of order ~ of first kind is given by $J_{\sim}(x) = \left(\frac{x}{2}\right)^{\sim} \sum \frac{(-1)^m}{m!\Gamma(m + \sim + 1)}\left(\frac{x}{2}\right)^{2m}$.

63. Show that a function ¤ is a solution of initial value problem $y' = f(x, y), y(x_0) = y_0$ on an interval I iff it's a solution of integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)dt$ on I.

64. Find the surface orthogonal to one parameters system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2, z = 0$.

PG–245 **MMS-15**

M.Sc. DEGREE EXAMINATION JUNE 2009.

First Year

(AY 2006–07 batches onwards)

Mathematics

ALGEBRA

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

65. Show that the set of all even permutations is a normal subgroup of S_n.

66. If $o(G) = p^n$ where p is a prime number show that $Z(G) \neq \{e\}$.

67. Let A be an ideal of an Euclidean ring R. Show that there exists $a_0 \in A$ such that A consists exactly all $a_0 x$ where $x \in R$.

68. State and prove Fermat's theorem.

69. If $\{V_1, V_2, ..., V_n\}$ in a vector space V has W as linear span and if $V_1, V_2, ...V_k$ are linearly independent, show that we can find a subset of $V_1, V_2, ...V_n$ of the form $\{V_1, V_2., V_k, V_{i_1}, V_{i_2} \cdots V_{i_r}\}$ consisting of linearly independent elements whose linear span is also W.

70. Show that the vectors

    $\{(1, 1, 0, 0), (0, -1, -1, 0), (0, 0, 0, 3)\}$ are linearly independent.

71. State and prove factor theorem.

72. If $T \in A(V)$ is nil potent show that $G_0 I + a_1 T + ... + a_n T^n$ is invertible if $a_0 \neq 0$ and $a_i \in F$.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

73. Let H and K be two subgroups of a group G. Show that HK is a subgroup of G if and only if $HK = KH$.

74. Show that the number of p-Sylow subgroups in G for any given prime p is of the form $1 + kp$.

75. Show that an ideal $A = (a_0)$ is a maximal ideal of an Euclidean ring R iff a_0 is a prime in R.

76. State and prove unique factorization theorem.

77. If V and W are vector spaces of dimensions m and n respectively show that $Hom(U, V)$ is of dimension mn.

78. Show that $a \in K$ is algebraic over a field F if and only if $F(a)$ is a finite extension of F.

79. If F is of characteristic 0 and if a, b algebraic over F, show that there exists an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.

80. (a) If V is finite dimensional over F show that $T \in A(V)$ is singular if and only if there exists $v \neq 0$ such that $T(v) \neq 0$.

(b) If ª is a characteristic root of $T \in A(V)$, show that for any polynomial $f(x) \in F[x]$, $f($ª$)$ is a characteristic root of $f(T)$.

PG–246 **MMS–16**

M.Sc. DEGREE EXAMINATION —
JUNE, 2009.

(AY 2006–07 batches onwards)

First Year

Mathematics

REAL ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

81. Show that a set E is open if and only if its complement is closed.

82. Show that the subsequential limits of a sequence $\{p_n\}$ in a metric space X form a closed subset of X.

83. Show that composition of two continuous functions is continuous.

84. Suppose f is continuous on $[a, b]$, $f'(x)$ exists at some point $x \in [a, b]$, g is defined on an interval I which contains the range of f and g is differentiable at the point $f(x)$. If $h(t) = g(f(t))$ for $a \leq t \leq b$, show that h is differentiable at x.

85. Show that $f \in R(\sim)$ on $[a, b]$ if and only if for every $\in > 0$ there exists a partition p such that $U(p, f, \sim) - L(p, f, \sim) < \in$.

86. State and prove Wierstrass test for uniform convergence.

87. Show that a linear operator A on a finite dimensional vector space X is one-to-one if and only if the range of A is all of X.

88. Show that a linear operator A on R^n is invertible if and only if $\det[A] \neq 0$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

89. Show that every nonempty perfect set in R^k is uncountable.

90. Let Σa_n be a series of real numbers which converges, but not absolutely. Let $-\infty \leq \sim \leq \quad \leq \infty$. Show that there exists a rearrangement Σa_n^1 with partial sums s_n^1 such that

$$\underset{n \to \infty}{Lim} \inf s_n^1 = \sim ; \quad \underset{n \to \infty}{Lim} \sup s_n^1 = \quad .$$

91. Let E be a non compact set in R'. Show that

(a) There exists a continuous function on E which is not bounded.

(b) There is a continuous and bounded function on E which has no maximum.

If E is bounded show that there exists a continuous function on E which is not uniformly continuous.

92. State and prove L'Hospital's rule.

93. Let f be a bounded function on $[a, b]$ and $\sim$ increases monotonically such that $\sim' \in R$ on $[a, b]$. Show that $f \in R(\sim)$ if and only if $f\sim' \in R$ and $\int_a^b f\, d\sim = \int_a^b f(x) \sim'(x)\, dx$.

94. Let $\{f_n\}$ be a sequence of functions differentiable on $[a, b]$ such that $\{f_n(x_0)\}$ converges for some point $x_0 \in [a, b]$. If $\{f_n'\}$ converges uniformly on $[a, b]$ show that $\{f_n\}$ converges uniformly on $[a, b]$ to a function f and $f'(x) = \underset{n\to\infty}{Lim} f_n'(x)$.

95. State and prove Stone Weirstrass theorem.

96. State and prove Implicit function theorem.

PG-247	**MMS–17**

M.Sc. DEGREE EXAMINATION
JUNE 2009.

First Year

Mathematics

(AY 2006–07 batch onwards)

COMPLEX ANALYSIS AND NUMERICAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

97. Prove that the real and imaginary parts of an analytic function are harmonic functions.

98. Find an analytic function whose real part is $u = 2xy + 2x$.

99. State and prove Cauchy's integral formula.

100. Compute $\int_C \frac{e^z \, dz}{(z-1)\ (z+2)}$ where C is the circle $|z-1| = 1$.

101. Using analytic method find the approximate value of the root of equation

$$3x - \sqrt{1+\sin x} = 0.$$

102. If $f(x) = x^3 - 2x$, find the divided differences of order one, two and third for the arguments 1, 2, 4 and 7.

103. Solve the following systems of equations by Cramer's rule :

$$x + 2y + 3z = 10,\ \ 2x - 3y + z = 1,\ \ 3x + y - 2z = 9.$$

104. Find the value of y for $x = 0.1$ by Picard method, given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

105. Let $f(z) = u + iv$ be analytic in D. Then prove that

(a) if u is constant then f is constant

(b) if $|f|$ is constant then f is constant.

106. If a bilinear transformation has exactly two fixed points z_1 and z_2, then for some nonzero constant k, prove that they satisfy the equation

$$\frac{w - z_1}{w - z_2} = k \cdot \frac{z - z_1}{z - z_2}$$

107. Suppose ¤(w) is continuous on the arc ¥ . Then prove that the function $F_n(z) = \int_{¥} \frac{¤\ (w)\ dw}{(w-z)^n}$ is analytic in each of the regions determined by ¥ and $F_n^{(1)}(z) = n\ F_{n+1}(z)$.

108. State and prove Taylor's theorem for an analytic function.

109. Applying Newton-Raphson method find the real root of the equation $x^3 - 3x - 5 = 0$.

110. Solve, by the method of factorization

$$\begin{aligned} x+2y+3z &= 14, \\ 2x+3y+4z &= 20, \\ 3x+4y+z &= 14. \end{aligned}$$

111. Given that $u_{16}=39$, $u_{18}=85$, $u_{22}=151$, $u_{24}=264$ and $u_{26}=388$. Compute u_{20} using the difference table.

112. By Taylor series method, find the value of y at $x=0.1$ from $\frac{dy}{dx}=x^2y-1$, $y(0)=1$.

PG–248	**MMS-18**

M.Sc. DEGREE EXAMINATION – JUNE 2009.

(AY 2006-07 batch onwards)

First Year

Mathematics

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

Each question carries 5 marks.

113. Define probability set function. State the axioms of probability.

114. Find the marginal density function for X and Y if the joint p.d.f. is

$$f(x,y) = \begin{cases} 4xy; & 0<x<1,\ 0<y<1 \\ 0; & \text{otherwise} \end{cases}$$

115. A continuous random variable X has p.d.f.

$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2 & ; \quad -3 \le x \le -1 \\ \frac{1}{16}(6-2x^2) & ; \quad -1 \le x < 1 \\ \frac{1}{16}(3-x) & ; \quad 1 \le 0 \le 3 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Find the mean of the random variable.

116. If a random variable has a binomial population with parameters n and p. Show that the sample proportion $\frac{X}{n}$ is an unbiased estimator of p.

117. Show that the sample variance $s^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$ is a consistent estimator of the population variance $±^2$ where $X_1, X_2, \cdots, X_n$ is a random sample from a normal population $N(¯, ±^2)$.

118. Define conditional probability. Hence define mutually independent events.

119. What is point estimation? How is it different from interval estimation?

120. Write a note on Bayesian estimation.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

121. State and prove Chebychev's inequality.

122. Find the moment generating function of the Poisson distribution. Hence find its mean and variance.

123. Derive the density function of the ¥ -distribution.

124. Find the critical region of the likelihood ratio test for testing the null hypothesis $H_0 : « = «_0$ against the composite alternative hypothesis $H_1 : « \neq «_0$ on the basis of a random sample of size n from a normal population with the known variance $±^2$.

125. State and prove central limit theorem.

126. Derive the confidence interval for the difference of two proportions for a normal population.

127. State and prove Rao-Blackwell theorem.

128. State and prove Rao-Cramer inequality.

PG–241	**MMS–21**

M.Sc. DEGREE EXAMINATION —
JUNE, 2009.

(AY 2005–06 and CY–2006 batches only)

Second Year

Mathematics

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

129. Find the value of the constant C if $f(x) = c\left(\frac{2}{3}\right)^x; \; x = 1, 2, 3 \ldots$; 0 ; elsewhere is a p.d.f of a random variable X.

130. A factory manufacturing television has four units A, B, C, D. The units manufacture 15%, 20%, 30%, 35% of the total output respectively. It was found that out of their outputs 1%, 2%, 2%, 3% are defective. A television is chosen at random from the output and found to be defective. What is the probability that it came from unit D?

131. Let X be a random variable with probability distribution $p(x) = \frac{1}{8}\, {}_3C_x; \; x = 0, 1, 2, 3$. Find the moment generating function and hence find its mean and variance.

132. Let $X_1, X_2 \ldots X_n$ be mutually stochastic independent random variables having respectively, the normal distribution $n(«_1, \pm_1^2), n(«_2, \pm_2^2), \ldots n(«_n, \pm_n^2)$. Let $Y = k_1X_1 + k_2X_2 + \ldots + k_nX_n$ where $k_1, k_2 \ldots k_n$ are constant. Prove that Y is normally distributed with mean $k_1«_1 + k_2«_2 + \ldots + k_n«_n$ and $k_1^2\pm_1^2 + k_2^2\pm_2^2 + \ldots + k_n^2\pm_n^2$.

133. Let Z_n be $¡^2(n)$. Find the limiting distribution of the random variable $Y_n = \dfrac{Z_n - n}{\sqrt{2n}}$.

134. Let $X_1, X_2 \ldots X_n$ be a random sample from the exponential distribution with the density function $f(x) = \begin{cases} a e^{-ax} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$ Find the maximum likelihood estimator of a.

135. In a certain political campaign, 185 out of 351 voters favour particular candidate. Find the 95% confidence interval for the fraction p of the voting population who favour this candidate.

136. Define (a) consistent estimator and (b) unbiased estimator.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

137. State and prove Chebychev's inequality.

138. If (X, Y) is a continuous random variable with p.d.f. $\begin{aligned} f(x,\ y) &= e^{-x-y}; \ x > 0 > y \\ &= 0 \quad ; \ \text{elsewhere,} \end{aligned}$ find the correlation coefficient between X and Y.

139. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and variance of the distribution.

140. Derive t–distribution.

141. Let $F_n(y)$ denote the distribution function of a random variable Y_n whose distribution depends upon the positive integer n. Let c be a constant which does not depend upon n. The random variable Y_n converges stochastically to the constant c if $\forall \in > 0$, $\lim\limits_{n \to \infty} P(|Y_n - c| < \in) = 1$. Prove.

142. State and prove Neymann–Factorisation theorem.

143. State and prove Rao–cramer inequality.

144. State and prove Neymann–Pearson theorem.

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PG–242	**MMS–22**

M.Sc. DEGREE EXAMINATION –
JUNE 2009.

(AY 2005-06 and CY 2006 batches only)

Second Year

Mathematics

OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

145. Explain the standard form for linear programming problem. Write the standard form of the following.

Maximize $z = 5x_1 + 3x_2$

subject to the constraints

$$x_1 + x_2 \leq -4$$
$$5x_1 + 2x_2 \leq 10$$
$$3x_1 + 8x_2 \leq 12$$
and $x_1, x_2 \geq 0$.

146. Explain the basic idea of the interior point used in Karmakar algorithm.

147. What is meant by the term critical activities and why is it necessary to know about them?

148. Obtain the functional equation for maximizing

$$Z = g_1(x_1) + g_2(x_2) + \cdots + g_n(x_n)$$

subject to the constraints

$$x_1 + x_2 + \cdots + x_n = c \text{ and } x_1, x_2, \cdots x_n \geq 0.$$

149. For the game with the following pay-off matrix, determine the optimum strategies and the value of the game.

$$\begin{array}{cc} & P_2 \\ P_1 & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \end{array}$$

150. What do you understand by (a) queue discipline (b) queue size?

151. Define Hessian matrix. Write the sufficient condition for a stationary point x_0, to be an extremum.

152. Show that the problem $Max\ Z = x_1 x_2$ is separable.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

153. Use simplex method to
Maximize $Z = 4x_1 + 3x_2$
Subject to the constraints

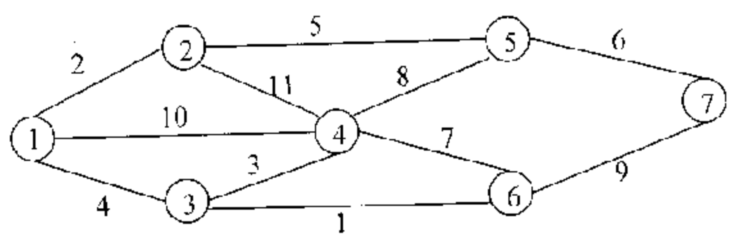
$$2x_1 + x_2 \leq 1000$$
$$x_1 + x_2 \leq 800$$
$$x_1 \leq 400$$
$$x_2 \leq 700$$

and $x_1, x_2, x_3 \geq 0$.

154. Carry out three iterations of Karmakar's algorithm for the following problem :
Maximize $z = x_1 - 2x_2$
subject to

$$x_1 - 2x_2 + x_3 = 0$$
$$x_1 + x_2 + x_3 = 1$$
$$x_1, x_2, x_3 \geq 0.$$

155. Determine the shortest-route of the network using Acyclic algorithm.



156. Using dynamic programming techniques solve :
Maximize $Z = x_1^2 + 2x_2^2 + 4x_3$
subject to the constraints

$$x_1 + 2x_2 + x_3 \leq 8$$

and $x_1, x_2, x_3 \geq 0$.

157. For the following pay-off table transform the zero sum game into an equivalent linear programming problem and solve it by simplex method.

		Player Q		
		Q_1	Q_2	Q_3
	P_1	9	1	4
Player P	P_2	0	6	3

P$_3$	5	2	8

158. A television repairman finds that the time spends on his jobs has an exponential distribution with a mean of 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets a Poisson distribution approximately with an average of 10 per 8 hour day. (a) What is the repairman's expected idle time each day? (b) Find the average number of televisions waiting for service in the system.

159. Examine the function

$$f(x) = x_1 + 2x_3 + x_2x_3 - x_1^2 - x_2^2 - x_3^2$$

for extreme points.

160. Find the mean recurrence time for each state of the following Markov Chain $\begin{bmatrix} 0.1 & 0.9 \\ 0.7 & 0.3 \end{bmatrix}$.

PG–243 MMS–23

M.Sc. DEGREE EXAMINATION – JUNE 2009.

Second Year

(AY 2005–06 and CY – 2006 batches only)

Mathematics

GRAPH THEORY AND DATA STRUCTURES

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

161. Define graph and show that the number of vertices of odd degree in a graph is always even.

162. Define a connected graph. Show that a graph G is disconnected if its vertex set V can be partitioned into two non-empty disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in V_1 and the other in V_2.

163. (a) Prove that a closed walk of odd length contains a cycle.

(b) If G is not connected then prove that $\overline{G}$ is connected.

164. Define center of a tree and prove that every tree has either one or two centers.

165. Prove that an edge e of G is a cut edge of G if and only if e is contained in no circuit of G.

166. Define a planar graph and prove that K_5 and $K_{3,3}$ are not planar.

167. Write down the chromatic polynomial for the graph $K_4 - e$.

168. Give a procedure for the breadth first traversal algorithm.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

169. Define bipartite graph and show that a graph G with atleast two vertices is bipartite if and only if all its cycles are of even length.

170. Define Eulerian graph and prove that a connected graph G is Eulerian if and only if each vertex of G has even degree.

171. Define the operations (a) ring sum (b) decomposition (c) edge deletion (d) vertex deletion and (e) fusion and explain each one with example.

172. For any graph G, prove that $K(G) \leq K'(G) \leq ¢\,(G)$.

173. Prove that the graphs $K_{3,\,3}$ and K_5 cannot have dual.

174. Prove that G has a perfect matching if and only if $O\,(G-S) \leq |S|$ for all $S \subseteq V$.

175. State and prove the Vizing's theorem.

176. Let T be a 2-tree with k leaves. Then the height h of T satisfies $h \geq [\lg k]$ and the external path length $E(T)$ satisfies $E(T) \geq k \lg k$. The minimum values for h and $E\,(T)$ occur when all the leaves of T are on the same level or on two adjacent levels.

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M.Sc. DEGREE EXAMINATION —
JUNE, 2009.

(AY 2005–06 and CY 2006 batches only)

Second Year

Mathematics

FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

177. If M is a closed linear subspace of a normed linear spare N and 1 and T is the natural mapping of N onto N/M defined by $T(x) = x + M$, show that T is a continuous linear transformation for which $\|T\| \leq 1$.

178. State and prove Uniform boundedness theorem.

179. Let M be a closed liner subspace of a Hilbert space H, let $x \notin M$ and let d be the distance from x to M. Show that there is a unique vector $y_0 \in M$ such that $\|x - y_0\| = d$.

180. If A is a positive operator on a Hilbert space H show that $1 + A$ is non singular.

181. Let B be a basis for H and T an operator whose matrix relative to B is $[\sim_{ij}]$. Show that T is nonsingular if and only if $[\sim_{ij}]$ is non singular.

182. Let R be the radical of a Banach algebra A. Show that if $1 - xr$ is regular for $x \in A$ and $r \in R$, $1 - rx$ is also regular.

183. Let A be a commutative Banach Algebra show that the following conditions are equivalent

(a) $\|x^2\| = \|x\|^2$ for $x \in A$

(b) $r(x) = \|x\|$ for $x \in A$

(c) $\|\hat{x}\| = \|x\|$ for $x \in A$.

184. Let X and Y be nls and $F : X \to Y$ be linear. Show that F is compact if and only if for every bounded sequence (x_n) in X, $(F(x_n))$ contains a subsequence which converges in Y.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

185. Let M be a linear subspace of a normed linear space N and f be a functional defined on M. If $x_0 \notin M$ and if $M_0 = M + [x_0]$ show that f can be extended to a functional f_0 defined on M_0 such that $\|f_0\| = \|f\|$.

186. If T is an operator on a normed linear space N, show that its conjucate T^* is an operator on N^* and the mapping $T \to T^*$ is an isometric isomorphism of $B(N)$ into $B(N^*)$ which reverses products.

187. If $\{e_i\}$ is an orthonormal set in a Hilbert space H, and if $x \in H$ show that $x - \Sigma(x, e_i)e_i \perp e_j$ for each j.

188. (a) If T is an operator on Hilbert space H, show that T is normal if and only if its real and imaginary parts commute.

(b) Show that an operator T is unitary if and only if it is an isometric isomorphism of H onto itself.

189. Let T be a normal operator on a Hilbert space H with eigen values $a_1, a_2, \ldots, a_n$ and eigen spaces $M_1, M_2, \ldots, M_n$. Show that M_i's are pairwise orthogonal, each M_i reduces T and M_i's span H.

190. For each x in a Banach Algebra A, show that the spectral radius $r(x) = \lim \|x^n\|^{\frac{1}{n}}$.

191. State and prove Gelfand Neumark Theorem.

192. (a) Define compact linear map and show that collection of all compact linear maps is a subspace of $BL(X, Y)$.

(b) Show that the spectrum $^\circ(A)$ of a compact map A is countable.

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PG–249 **MMS–25**

M.Sc. DEGREE EXAMINATION – JUNE 2009.

(AY 2006-2007 batch onwards)

Second Year

Mathematics

TOPOLOGY AND FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

193. Let **B**,**B**' the bases for topologies τ_1 and τ_2 respectively. Show that τ_2 is finer than τ_1, if and only if for each x and each $B \in \mathbf{B}$ containing x there is a $B' \in \mathbf{B'}$ such that $x \in B' \subset B$.

194. Let $\{X_i : i \in I\}$ be a family of spaces and $A_i \subset X_i$. Show that in both product and box topologies $\prod \overline{A_i} = \overline{\prod A_i}$.

195. Show that finite product of connected spaces is connected.

196. State and prove extreme value theorem.

197. Show that subspace of a regular space is regular.

198. Let $T : N \to N'$ be a linear transformation from a normed linear spaced to another. Show that the following conditions are equivalent.

(a) T is continuous

(b) T is continuous at origin

(c) There exist a real number $K > 0$ such that $\| T(x) \| \leq K \| x \|$ for all $x \in N$.

199. State and prove uniform bondedness theorem.

200. If T is an operator on a Hilbert Space H such that $(T(x), x) = 0$ for all x, show that $T = 0$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

201. Show that the topologies on R^n in duced by the Euclidean metric and the square metric are the same as the product topology.

202. (a) Let $f : x \to y$ where x is a metric space and y is a topological space. Show that f is continuous if and only if $x_n \to x$ implies $f(x_n) \to f(x)$.

(b) Let $A \subset X$ and A' be the set of all limit points of A. Show that $\overline{A} = A \cup A'$.

203. (a) Show that the ordered square I^2 with ordered topology is connected but not path connected.

(b) If X is locally path connected show that the components and the path components of X are the same.

204. Let X be a locally compact Hausdorff space. Show that there exists a space Y such that X is a subspace of Y, $Y - X$ consists of a single point and Y is compact Hausdorff.

205. State and prove Uryshon's lemma.

206. State and Prove Hahn Banach theorem.

207. Let H be a Hilbert space and f be an arbilrary functional. Show that there exists a unique $y \in H$ such that $f(x) = (x, y)$.

208. (a) Show that an operator T is unitary if and only if it is an isometric isomorphism.

(b) Show that an operator T is normal if and only if its real and imaginary parts commute.

PG–250	**MMS–26**

M.Sc. DEGREE EXAMINATION —
JUNE, 2009.

(AY 2006–07 batch onwards)

Second Year

Mathematics

OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

209. Obtain the dual of the following linear programming problem

Maximize $Z = x_1 - 2x_2 + 3x_3$

subject to the constraints

$$-2x_1 + x_2 + 3x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 = 1$$

and $x_1,\ x_2,\ x_3 \geq 0$.

210. Explain the linear goal programming.

211. Explain the concept of dynamic programming and the relation between linear programming and dynamic programming approach.

212. Two players A and B match coins. If the coins match, then A wins two units of value, if the coins do not match, then B wins 2 units of value. Determine the optimum strategies for the players and the value of the game.

213. If the two person, zero-sum game

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 0 & 2 \\ -1 & 4 \end{bmatrix} \end{array}$$

Strictly determinable and fair? Justify.

214. Explain the $(M/M/1):(GD/\infty/\infty)$ queuing model.

215. State the sufficient conditions in Lagrange's multiple method.

216. Show that the problem $Max\ z = x_1\, x_2$ is separable.

SECTION B — ($5 \times 10 = 50$ marks)

Answer any FIVE of the following.

217. Use simplex method to maximize $z = 5x_1 + 4x_2$

subject to the constraints :

$$4x_1 + 5x_2 \leq 10$$
$$8x_1 + 3x_2 \leq 12$$
$$3x_1 + 2x_2 \leq 9$$
$$x_1,\ x_2 \geq 0.$$

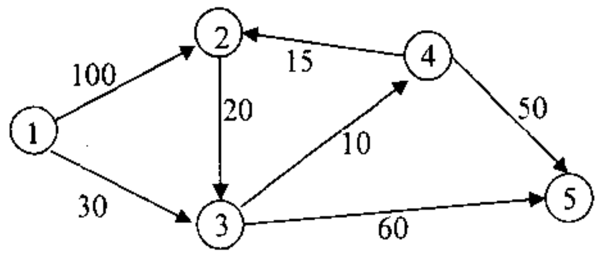
218. Carry out three iterations of Karmarkar's algorithm for the following problem :

Maximize $Z = x_1 - 2x_2$

subject to

$$x_1 - 2x_2 + x_3 = 0$$
$$x_1 + x_2 + x_3 = 1$$
$$x_1,\ x_2,\ x_3 \geq 0.$$

219. Solve the network by cyclic algorithm



220. Solve the following linear programming problem by dynamic programming

Maximize $Z = 8x_1 + 7x_2$

subject to the constraints

$$2x_1 + x_2 \leq 8$$
$$5x_1 + 2x_2 \leq 15$$
$$x_1,\ x_2 \geq 0.$$

221. Use Branch and Bound method to solve the following their programming problem

Maximize $Z = 5x_1 + 4x_2$

subject to

$$x_1 + x_2 \leq 5$$
$$10x_1 + 6x_2 \leq 45$$

and $x_1,\ x_2 \geq 0.$

222. In a shop there is only one salesman at the counter. 9 customers arrive on an average every
5 minutes, while the salesman can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrivals and exponential distribution for service rate. Find (a) average number of customers in the system, (b) average number of customers in the queue, (c) average time a customer spends in the system, and (d) average time a customer spends in the queue.

223. Obtain the necessary conditions for the optimum solution of the following problem

Minimize $f(x_1,\ x_2) = 3e^{2x_1+1} + 2e^{x_2+5}$

Subject to the constraint

$g(x_1,\ x_2) = x_1 + x_2 - 7 = 0$.

224. Solve the following geometric programming problem

Minimize $Z = 7x_1x_2^{-1} + 3x_2x_3^{-2} + 5x_1^{-3}\, x_2\, x_3 + x_1x_2x_3$.

PG–252	**MMS-28**

M.Sc. DEGREE EXAMINATION — JUNE 2009.

Second Year

(AY 2006–07 batch onwards)

Mathematics

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

225. Discuss the solution of the differential equation $L(y) = y'' + a_1 y' + a_2\ y = 0$, where a_1, a_2 are constants.

226. Solve the equation $2y'' - 5y' + 3y = 0$.

227. If $P_n(x)$, $P_m(x)$ are Legendre polynomials, prove that $\int_{-1}^{1} P_n(x)\, P_m(x)\, dx = 0,\ n \neq m$.

228. Determine the regular singular point of the equation
$x^2 y'' + (x + x^2) y' - y = 0$.

229. Let Φ be a fundamental matrix of the system of equations $y' = A(x)\ y$ and let C be a constant
non-singular matrix. Prove that ΦC is also a fundamental matrix of above system.

230. Find a fundamental matrix of the equation

$$y' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{bmatrix} \cdot y$$

231. Solve the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = \frac{\partial^4 z}{\partial x^2 \partial y^2}$.

232. Derive the elementary solution of the Laplace equation $\nabla^2 \cdot \ = 0$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

233. Let ¤ be any solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval containing a point x_0 . Then prove that for all x in I ,

$\|¤\ (x_0)\|\ e^{-K|x-x_0|} \leq \|¤\ (x)\| \leq \|¤\ (x_0)\|\ e^{K\ |x-x_0|}$ where $K = |a_1| + |a_2|$.

234. Find the solution of the initial value problem $y'' - 2y' + y = 2x$, $y(0) = 6$, $y'(0) = 2$.

235. Derive the power series solution of the equation $\left(1 - x^2\right)y'' - 2xy' + \sim (\sim + 1)\ y = 0$, where $\sim$ is a constant.

236. Determine the solution of Bessel equation $x^2 y'' + xy' + \left(x^2 - \sim^2\right)y = 0,$ when $\sim\ = 0$.

237. State and prove the existence and uniqueness theorem for the IVP $y' = A(x)y,\ y(x_0) = y_0\ ,\ x,\ x_0\ £\ I$.

238. Let $f(x)$ be periodic with period w . Let A be an $n \times n$ constant matrix. Prove that a solution of $y' = Ay + f(x)$ is periodic of period w if and only if $y(0) = y(w)$.

239. Reduce the equation

$\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ into its canonical form.

240. State and prove Kelvin's inversion theorem.