PG–931 **MMS-11**

M.Sc. DEGREE EXAMINATION – JUNE 2010.

First Year

Mathematics

(AY – 2005-06 and CY – 2006 batches only)

ALGEBRA

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Show that subgroup of a cyclic group is cyclic.

2. State and prove Cayley's theorem.

3. Let R be a ring such that $a^2 = a$ for all $a \in R$. show that R is commutative.

4. Let R be an Euclidean ring, show that any two elements $a,b \in R$ have a greatest common divisor d and $d =$ ª $a +$ «b where ª,« $\in R$.

5. Show that intersection of two subspaces of a vector space is also a subspace. Is this result for union? Justify your answer.

6. State and prove Remainder theorem.

7. Find the splitting field of $x^4 - 5x^2 + 6$ over Q.

8. If $f(x)$ and $g(x)$ in $F(x)$ have a non-trivial common factor in K[x] where K is an extension of F show that $f(x)$ and $g(x)$ have a non trivial common factor in $F(x)$ also.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let H and K be 2 subgroups of a group G. Show that HK is a subgroup of G if and only if HK = KH.

10. Let G be a finite abelian group and $p/o(G)$. Where p is a prime number. Show that $a \neq e$ in G such that $a^p = e$.

11. If U is an ideals of the ring R show that R/U is a ring and is a homomorphic image of R.

12. Show that an ideal $A = (a_{\circ})$ is a maximal ideal of the Euclidean ring R iff $a_{\circ}$ is a prime in R.

13. (a) If V is finite dimensional over F and if $u_1, u_2 ...., u_m \in V$ are linearly independent, show that we can find vectors $u_{m+1}, ..., u_{m+r}$ in V such that $u_1, u_2, ...u_m, u_{m+1}, ..., u_{m+r}$ is a basic of V.

    (b) Prove that the vectors $\{(1,1,0,0), (0,-1,-1,0),$ $(0,0,0,3)\}$ are linearly independent.

14. If L is a finite extension of K and if K is a finite extension of F. Show that L is a finite extension of F and [L : F] = [L : K] {K : F].

15. Let ^{2} be a homomorphism from a field F onto F^1 where ${}^{2}(a) = a^1$, and ${}^{2\,*} : F[x] \to F^1[t]$ such that ${}^{2\,*}(a_0 + a_1 x + ... + a_n x^n) = a_0^1 + a_1^1 t + ... + a_n^1 t^n$. Show that there is an isomorphism

$${}^{2\,**} : \frac{F[x]}{\langle f(x) \rangle} \to \frac{F^1[t]}{\langle f^1(t) \rangle} \text{ such that}$$

$${}^{2\,**}\left(x + \langle f(x) \rangle\right) = t + \langle f^1(t) \rangle \; where \; {}^{2\,*}(f(x) = f^1(t).$$

16. If K is a finite extension of F show that G (K : F) is a finite group and $0(G(K : F) \leq [K : F]$.

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PG–932	**MMS-12**

M.Sc. DEGREE EXAMINATION –
JUNE 2010.

First Year

(AY – 2005-06 and CY – 2006 batches only)

Mathematics

REAL AND COMPLEX ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

17. Let $\{S_n\}$be monotonic. Prove that $\{S_n\}$ converges if and only if it is bounded.

18. State and prove the ratio test of convergence of series.

19. If $\sum a_n$ converges absolutely, prove that $\sum a_n$ converges.

20. Let f be a non-empty set in R. Prove that there is a continuous function on E which is not bounded.

21. Compute $\left(\frac{-1+i\sqrt{3}}{2}\right)^6$.

22. Show that the real and imaginary parts of an analytic function are harmonic functions.

23. State and prove Cauchy's integral formula.

24. State and prove Hurwitz theorem.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

25. If $\{p_n\}$ is a sequence in a compact metric space X, then prove that some subsequence of $\{p_n\}$ converges to a point of X.

Prove that every bounded sequence in R^K contains a convergent subsequence.

26. Let f be a continuous 1-1 mapping of a compact metric space X onto a metric space Y. Then show that the inverse mapping f^{-1} defined on Y by $f^{-1}(f(x)) = x$ for all $x \in X$ is a continuous mapping of Y onto X.

27. State and prove Taylor's theorem for a real function.

28. Show that $f \in R(\sim)$ on $[a,b]$ if and only if for $\in > 0$, there exists a partition P satisfying

$U(P,f,\sim)-L(P,f,\sim)<\in$.

29. Consider the power series $\sum_{n=0}^{\infty} a_n z^n$. Then there exists a number R, $0 \le R \le \infty$, for $|z|<R$, prove that the sum of the series is an analytic function.

30. Prove that a metric space is compact if and only if every infinite sequence has a limit point.

31. Define the index of a point 'a' with respect to the curve ¥ and show that n(¥, a) is a multiple of 2®i.

32. State and prove Cauchy's residue theorem.

PG–933 **MMS-13**

M.Sc. DEGREE EXAMINATION –
JUNE 2010.

First Year

(AY – 2005-06 and CY – 2006 batches only)

Mathematics

TOPOLOGY AND MEASURE THEORY

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

33. If $\{^{2}{}_{\sim}\}$ is a collection of topologies on X, then show that $\cap^{2}{}_{\sim}$ is a topology on X? Does $\cup^{2}{}_{\sim}$ a topology on X?

34. Define uniform convergence of a sequence of functions and state and prove the uniform limit theorem.

35. Define uniformly continuous function and state and prove the uniform continuity theorem.

36. Define normal space and regular space also prove that every metrizable space is normal.

37. Define Lebesgue measure and prove that the Lebesgue measure is countably additive.

38. If f and g are nonnegative measurable functions, then prove that

$$\int_E f+g=\int_E f+\int_E g.$$

39. State and prove the Fatou's lemma (for general measure).

40. State and prove the Lebesgue Decomposition theorem.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

Each question carries 10 marks.

41. (a) Prove that the topologies on $\mathbf{R}^n$ induced by the Euclidean metric d and the square metric are same as the product topology on $\mathbf{R}^n$. (7)

(b) Prove that $\mathbf{R}^w$ in the box topology is not metrizable. (3)

42. Let X be a simply ordered set having the least upper bound property. In the order topology, prove that each closed interval in X is compact.

43. (a) Define limit point compact and prove that compactness implies limit point compactness. (4)

(b) Let X be metrizable space then prove that X is sequentially compact implies X is compact. (6)

44. State the Tietze extension theorem and sketch the proof for the same.

45. (a) Prove that outer measure of an interval is its length. (7)

(b) Prove that the Lebesgue outer measure m* is countably subadditive. (3)

46. Let f be defined and bounded on a measurable set E with mE finite. Prove that

$$\inf_{f \le \cdot} \int_E \cdot\ (x)dx = \sup_{f \ge \ddot{}} \int_E \text{¤}(x)dx$$

for all simple functions ¨ and · , it is necessary and sufficient that f be measurable.

47. (a) Let (X, B) be a measurable space, $\langle «_n \rangle$ be a sequence of measures that converge setwise to a measure « , and $\langle f_n \rangle$ a sequence of nonnegative measurable functions that converge pointwise to the function f . Then prove that

$$\int f\, d« = \underline{\lim} \int f_n\, d«_n \, . \qquad (6)$$

(b) State and prove the Hahn Decomposition theorem. (4)

48. State the Fubini's theorem and prove the result by just stating the lemmas and proposition which you use.

PG–934	MMS-14

M.Sc. DEGREE EXAMINATION – JUNE 2010.

First Year

Mathematics

(AY – 2005-06 to CY – 2006 batches only)

NUMERICAL METHODS AND DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

49. Prove that Newton's method has second order convergence.

50. Find eigen values and eigen vectors of the matrix $\begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$.

51. Solve the equations :

$2x + 3y - z = 5$

$4x + 4y - 3z = 3$

$2x - 3y + 2z = 2$, by Cramer's rule.

52. Evaluate $\int_0^1 \frac{dx}{1+x}$, using Trapezoidal rule.

53. Find all solutions of $x^3 y''' + 2x^2 y'' - xy' + y = 0$ $(x > 0)$

54. Prove that $\sqrt{x} \;\; J_{1/2}(x) = \sqrt{\frac{2}{\circledR}} \;\sin x$.

55. Compute the first Two successive approximations $^\circ{}_0$ and $^\circ{}_1$ for $y' = x^2 + y^2, y(0) = 0$.

56. Find the complete integral of $p + q = pq$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

57. Using Aitken's Δ^2 method, solve the equation $2x = \cos\, x + 3$.

58. Find roots of the equation $x^3 - 2x + 2 = 0$, using Graeffe's root squaring method.

59. Using Runge-Kutta method (Fourth order) solve the equation $\frac{dy}{dx} = \sqrt{x+y}$ at $x = 0.8$ given, $y(0.4) = 0.41$, taking $h = 0.2$.

60. Develop the function $f(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x}\right)$ in a series of Chebychev's polynomials.

61. Verify that the function ¤$(x) = x^3 (x > 0)$ satisfies the equation $x^2 y'' - 7xy' + 15y = 0$ and also find a second independent solution.

62. Discuss the solutions of Euler equation $x^2 y'' + axy' + by = 0$, a, b are constants.

63. Solve the differential equation $y' = \frac{2x + 3y + 1}{x - 2y - 1}$.

64. Find complete integral of the equation $p^2 x + q^2 y = z$.

M.Sc. DEGREE EXAMINATION – JUNE 2010.

Second Year

(AY – 2005-06 and CY – 2006 batches only)

Mathematics

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

65. Find the value of C if $f(x) = \begin{cases} Cx\,e^{-x} ; 0 < x < \infty \\ 0 \quad ;\text{otherwise} \end{cases}$ is a probability density function of a random variable X.

66. A company has 3 machines M_1, M_2, M_3 which produce 20%, 30% and 50% of the products respectively. Their respective defective percentages are 7, 3 and 5. One product is chosen from these and inspected. It is defective. What is the probability that it has been made by machine M_3?

67. Find the moment generating function of a binomial distribution. Hence deduce its mean.

68. Let X have the probability density function $f(x) = \begin{cases} \left(\frac{1}{2}\right)^x ; x = 1, 2, 3 \ldots \\ 0 \quad ;\text{elsewhere.} \end{cases}$

Find the probability density function of $Y = X^3$.

69. Let $\overline{X}_n$ denote the mean of a random sample of size n from a distribution that has a mean « and positive variance $\frac{\pm^n}{n}$. Show that $\overline{X}_n$ converges stochastically to « if $\pm^2$ is finite.

70. Let $X_1, X_2, \ldots X_n$ be a random sample from the point binomial distribution with parameter p. Find the maximum likelihood estimator of p.

71. Let X be the length of life of a 6-watt light bulb marketed by a certain manufacturer of light bulbs. Assume that the distribution of X is $N(«, 1296)$. If a random sample of $n = 27$ bulbs were tested until they burned out, yielding a sample mean of $\overline{x} = 1478$ hours, find the 95% confidence interval for «.

72. Define Null and Alternate hypothesis.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

73. (a) State and prove Chebychev's inequality.

(b) Given that « = 0 and ± = 1, use Chebychev's inequality to find how much of the probability of X lies between 2 units of the mean?

74. If (X,Y) is a continuous random variable with p.d.f. $\begin{aligned} f(x,y) &= e^{-x-y}, x>0>y \\ &= 0 \quad , \text{elsewhere} \end{aligned}$

find the correlation coefficient between x and y.

75. Let X be normally distributed with mean 8 and standard deviation 4. Find (a) $P(5 \le X \le 10)$,
(b) $P(10 \le X \le 15)$, (c) $P(X \ge 15)$ (d) $P(X \le 5)$.

76. Derive the probability function of the t-distribution.

77. State and prove Neymann-Factorisation theorem.

78. State and prove Rao-Cramer inequality.

79. Let $F_n(y)$ denote the distribution function of a random variable Y_n whose distribution depends upon the positive integer n. Let c be a constant which does not depend upon n. Prove that the random variable Y_n converges stochastically to the constant C if for every $\in > 0$, $\lim\limits_{n \to \infty} P(|Y_n - C| < \in) = 1$.

80. Let X have a probability of the form $\begin{aligned} f(x,^{-}) &= ^{-} x^{^{-}-1} \; ; 0 < x < 1 \\ & 0 \quad ; \text{elsewhere where} \end{aligned}$

$^{-} \in \{^{-}/^{-} = 1,2\}$. To test the simple hypothesis $H_0 : ^{-} = 1$ against the alternative hypothesis $H_1 : ^{-}{}_1 = 2$. Use a random sample X_1, X_2 of size $n = 2$ and define the critical region to be $C = \left\{ (x_1, x_2) \Big/ \frac{3}{4} \le x_1, x_2 \right\}$. Find the power function of the test.

PG–936	**MMS-22**

M.Sc. DEGREE EXAMINATION – JUNE 2010.

Second Year

(AY – 2005-06 and CY – 2006 batches only)

Mathematics

OPERATIONS RESEARCH

Time : 3 hours　　　　Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

81. Obtain the dual of the following linear programming problem

    Minimize $Z = 4x_1 + 6x_2 + 18x_3$

    Subject to the constraints

    $x_1 + 3x_2 \geq 3$

    $x_2 + 2x_3 \geq 5$

    and $x_1, x_2, x_3 \geq 0$.

82. Explain the linear goal programming.

83. Explain the following terms in PERT/CPM.

    (a) Earliest occurrence time

    (b) Latest occurrence time.

84. Is the two person, zero-sum game

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \end{array}$$

strictly determinable and fair? Justify.

85. In a game of matching coins with two players, suppose A wins one unit of value, when there are two heads, wins noting when there are two tails and loses $\frac{1}{2}$ unit of value when there are one head and one tail. Determine the pay-off matrix, the best strategies for each player and the value of the game to A.

86. Explain the pure birth and pure death processes.

87. Write the Karush-Kuhn-Tucher (KKT) conditions for maximization problem.

88. Write the general form of a quadratic programming problem.

SECTION B — ($5 \times 10 = 50$ marks)

Answer any FIVE of the following.

89. Use simplex method to Maximize $Z = 3x_1 + 5x_2$

Subject to the constraints

$$3x_1 + 2x_2 \le 18,$$
$$x_1 \le 4$$
$$x_2 \le 6,$$
and $x_1, x_2 \ge 0.$

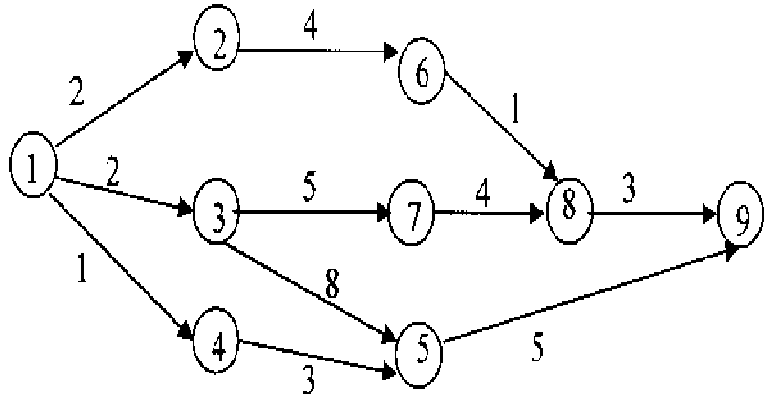
90. Apply the principal of duality to solve the following linear programming problem.

Minimize $Z = 2x_1 + 2x_2$

Subject to

$$2x_1 + 4x_2 \ge 1,$$
$$2x_1 + x_2 \ge 1$$
$$x_1 + 2x_2 \ge 1$$
and $x_1, x_2, x_3 \ge 0.$

91. Solve the network by cyclic algorithm.



92. Solve the following linear programming problem by dynamic programming approach.

Maximize $Z = 2x_1 + 5x_2$

Subject to the constraints

$2x_1 + x_2 \leq 43$

$2x_1 \leq 46$

$x_1, x_2 \geq 0$.

93. Use Branch and Bound method to solve the following inter programming problem.

Maximize $Z = 3x_1 + 2.5x_2$

Subject to

$x_1 + 2x_2 \geq 20$
$3x_1 + 2x_2 \geq 50$
and $x_1, x_2 \geq 0$ and integers

94. A departmental store has a single cashier. During the rush hours, customers arrive at a rate of 20 customers per hour. The average number of customers that can be processed by a cashier is 24 per hour. Assuming that the conditions for use of single channel queuing model apply. (a) What is the probability that the cashier is idle? (b) What is the average number of customers in the system? (c) What is the average time a customer spends in the system?

95. Determine x_1 and x_2 so as to maximize

$Z = 12x_1 + 21x_2 + 2x_1 x_2 - 2x_1^2 - 2x_2^2$

Subject to the constraints :

$x_2 \le 8,\ x_1 + x_2 \le 10$ and $x_1, x_2 \ge 0$.

96. Solve the following using separable programming.

Maximize $Z = x_1 + x_2^4$

Subject to

$$3x_1 + 2x_2^2 \le 9$$
$$x_1, x_2 \ge 0.$$

PG–937	**MMS-23**

M.Sc. DEGREE EXAMINATION –
JUNE 2010.

Second Year

(AY – 2005-06 and CY – 2006 batches only)

Mathematics

GRAPH THEORY AND DATA STRUCTURES

Time : 3 hours   Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

97. Distinguish between path and walk and prove a closed walk of odd length contains a cycle.

98. Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits.

99. Define a tree and prove that a tree with n vertices has $n-1$ edges.

100. Prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

101. Prove that in any connected plane (p, q) graph $(p \geq 3)$ with r faces $q \geq 3r/2$ and $q \leq 3p - 6$.

102. Prove that a matching M in G is a maximum matching if and only if G contains no M — augmenting path.

103. Define independent set and covering and show that $S \subseteq V$ is an independent set of a graph G with the set of vertices V if and only if $V - S$ is a covering.

104. Give a version of sequential search for linked lists.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

105. Define the operations (a) ring sum (b) decomposition (c) edge deletion (d) vertex deletion and (e) fusion and explain each one with example.

106. Define Eulerian graph and prove that a connected graph G is Eulerian if and only if each vertex of G has even degree.

107. (a) Prove that a connected graph with n vertices and $n - 1$ edges is a tree. (4)

(b) Prove that a graph G with n vertices, $n - 1$ edges and no circuit is connected. (3)

(c) Prove that in any tree with two or more vertices, there is at least two pendent vertices. (3)

108. (a) Prove that a graph G with $v \geq 3$ is 2– connected if and only if any two vertices of G are connected by at least two internally disjoint paths. (8)

(b) If G is a block with $v \geq 3$ then prove that any two edges of G lie on a common cycle.(2)

109. Prove that the graphs $K_{3,3}$ and K_5 cannot have dual.

110. Define covering and explain it with example and prove in a bipartite graph the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.

111. State and prove the Vizing's theorem.

112. (a) Define rooted tree, forest, ordered tree, orchard, subtree and give example for each
one. (5)

(b) Let S be any finite set of vertices. Prove that there is a one-to-one correspondence f from the set of orchards whose set of vertices is S to the set of binary trees whose set of vertices is S. (5)

PG–938 **MMS-24**

M.Sc. DEGREE EXAMINATION – JUNE 2010.

Second Year

Mathematics

(AY – 2005-06 and CY – 2006 batches only)

FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

113. Define a Banach space and show that norm is a continuous function.

114. Show that there is a norm preserving map from a normed linear space N into N^{**}.

115. State and prove Schwarz lemma.

116. If $\{e_1, e_2, ..., e_n\}$ is an orthonormal set in a Hilbert space H, show that for each $x \in H, \sum_{i=1}^{n} |(x, e_i)|^2 \leq \| x \|^2$.

117. If P is a projection on a Hilbert space H with range M and null space N, Show that $M \perp N$ if and only if p is self adjoint.

118. Show that in a Banach algebra, collection of all regular elements is open

119. If f_1 and f_2 are multiplicative functionals on a commutative Banach Algebra A with same null space M, Show that $f_1 = f_2$.

120. Let X, Y be n/s in which X is reflexive and $F : x \to y$ be linear. Show that F is compact if and only if it sends every week convergent sequence in X to a convergent sequence in Y.

SECTION B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

121. If M is a closed linear subspace of a Banach Space N, show that N/M is also a Banach space.

122. If N is a normed linear space, show that the closed unit sphere S^* in N^* is a compact Hausdorff space in the weak* topology.

123. (a) Show that parallelogram law is true in a Hilbert space.

(b) If B is a complex Banach space whose norm obeys the parallelogram law and if an inner product is defined by

$4(x, y) = \|x + y\|^2 - \|x - y\|^2 + i\,\|x + iy\|^2 - i\|x - iy\|^2$, then B is a Hilbert space. Prove this.

124. Let H be a Hilbert space and let f be an arbitrary functional in H^*. Show that there exists a unique vector $y \in H$ such that $f(x) = (x, y)$ for $x \in H$.

125. Show that two matrices in A_n are similar if and only if they are the matrices of a single operator on H relative to different bases.

126. Show that the mapping $x \to x^{-1}$ of the set of all regular elements G of a Banach Algebra onto G is a homomorphism.

127. Show that the Gelfand mapping $x \to \hat{x}$ is a norm decreasing homomorphism of A into $\boldsymbol{C}\,(m)$ with the following properties

(a) The image $\hat{A}$ of A is a sub algebra $\boldsymbol{C}\,(m)$ of which separates the points of on.

(b) The radical $R = \{x \in A : \hat{x} = 0\}$.

(c) $x \in A$ in regular if and only if $\hat{x}(M) \neq 0$ for every M.

128. If A is a compact operator and A^1 is the transpose, show that the finite dimensions of $Z(A - I)$ and $Z(A^1 - I)$ are equal.

PG–939 **MMS–15**

M.Sc. DEGREE EXAMINATION — JUNE, 2010.

First Year

Mathematics

ALGEBRA

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

129. Show that subgroup of a cyclic group is cyclic.

130. Let a group G be the inner direct product of $N_1, N_2, .., N_k$ and $T = N_1 \times N_2 \times .. \times N_k$. Show that T and G are isomorphic.

131. Let M be an ideal of a commutative ring R with identity show that M is maximal if and only if $\frac{R}{M}$ is a field.

132. Let p be a prime number and suppose that for some integer c relatively prime to p we can find integers x, y such that $cp = x^2 + y^2$. Show that p can be written as the sum of squares of two integers.

133. If V is an n-dimensional vector space over a field F, Show that V is isomorphic to $F^{(n)}$.

134. Let T be a module homomorphism. Show that T is an isomorphism if and only if $K(T) = 0$.

135. Let $f(x) \in F[x]$ be of degree $n \geq 1$. Show that there is an extension E of F of degree atmost n! in which $f(x)$ has n roots.

136. If V is a finite dimensional vector space over a field F, show that $T \in A(V)$ is regular if and only if T maps V onto V.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

137. Show that the set of all inner automorphisms of a group G, $I(G)$ is a group and $I(G)$ is isomorphic to G/Z where Z is the centre of G.

138. State and prove Sylow's theorem.

139. Show that $F[x]$ is an Euclidean ring.

140. (a) Let $C[0,1]$ be the ring of all real valued continuous functions defined on $[0,1]$. Show that $I=\left\{f\in C[0,1]:f\left(\frac{1}{2}\right)=0\right\}$ is a maximal ideal of $C[0,1]$.

(b) Show that every Euclidean ring posses a unit element.

141. If V and W are vector spaces over a field F of dimension m, n respectively, show that $Hom(V,W)$ is of dimension mn.

142. If L is a finite extension of K and if K is a finite extension of K and if K is a finite extension of F, show that L is a finite extension of F.

143. Let K be a normal extension of F and H be a subgroup of $G(K:F)$. Let $K_H = \{x \in K : \pm(x) = x \text{ for all } \pm \in K\}$. Show that $[K:K_H]=o(H)$ and $H = G(K:K_H)$.

144. If $W \subset V$ is invariant under T, show that T induces a linear transformation $\overline{T}$ on $\frac{V}{W}$ defined by $(V+W)\,\overline{T}=VT+W$. If T satisfies $q(x)\in F[x]$, show that $\overline{T}$ also satisfies it. If $P_1(x)$ is the minimal polynomial for T over F and if $p(x)$ is that for $\overline{T}$, show that $p_1(x)/p(x)$.

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PG–940	MMS–16

M.Sc. DEGREE EXAMINATION –
JUNE, 2010.

First Year

Mathematics

REAL ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

Each question carries 5 marks.

145. (a) If $x, y \in \mathbf{R}$ and $x > 0$, then prove that there exists a positive integer n such that $nx > y$.

(b) If $x, y \in \mathbf{R}$ and $x < y$, then prove that there exists a $p \in Q$ such that $x < p < y$.

146. If E is a subset of $\mathbf{R}^k$ such that every infinite subset of E has a limit point in E, then prove that E is closed and bounded.

147. State an sufficient condition for an alternating series to converge and prove the same.

148. "A function f of a metric space X into a metric space Y is continuous at point $x \in X$ if and only if $\{x_n\} \to x$ in X implies $\{f(x_n)\} \to f(x)$ in Y'' – Prove.

149. State and prove the generalized mean value theorem and give an example to show that the result is not true for complex valued functions.

150. Let $f \in R$ on $[a,b]$. For $a \le x \le b$, put $F(x) = \int_a^x f(t)dt$. Then prove that F is continuous on $[a,b]$, further if f is continuous at a point x_6 of $[a,b]$; then f is differentiable at x_0, and $F'(x_0) = f(x_0)$.

151. Give a sequence of function $\{f_n\}$ on $[a,b]$ such that $\{f_n\} \to f$ pointwise and $f_n \in R$ but $f \notin R$ on $[a,b]$.

152. State and prove the chain rule of differentiation of function of several variables.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

Each question carries 10 marks.

153. A sequence $\{x_n\}$ in $\mathbf{R}^k$ converges if and only if $\{x_n\}$ is a Cauchy sequence. Prove this.

154. State and prove the Riemann's rearrangement theorem.

155. (a) Let f and g be complex continuous functions on a metric space X. Then prove that $f + g$, fg and f/g are continuous on X.

(b) Let $f_1, f_2, \ldots, f_k$ he real functions on a metric space X, and let f be the mapping defined by

$f(x) = (f_1(x), \ldots, f_k(x)),\ x \in X$

Then prove that f is continuous if and only if each if the functions $f_1, \ldots, f_k$ is continuous.

(c) If $f, g : X \to \mathbf{R}^k$ are continuous then prove that $f + g$ and $f \circ g$ are continuous on X.

156. (a) State and prove the Taylor's theorem.

(b) Prove that $f \in R(\sim)$ on $[a,b]$ if and only if for every $£ > 0$ there exists a partition P such that

$U(P, f, \sim) - L(P, f, \sim) < £$.

157. State and prove the Stone-Weierstrass theorem.

158. Define e^x and show that e^x is well defined on $\mathbf{R}$ also prove

(a) e^x is continuous and differentiable for all x.

(b) $\left(e^x\right)' = e^x$

(c) e^x is strictly increasing function of x and $e^x > 0$

(d) $e^{x+y} = e^x \cdot e^y$

(e) $e^x \to \infty$ as $x \to \infty$ and $e^x \to 0$ as $x \to -\infty$

(f) $\lim\limits_{x \to \infty} x^n e^{-x} = 0$, for every n.

159. State and prove the Parseval's theorem.

160. (a) If X is a complete metric space, and if ¤ is a contraction of X into X, then prove that there exists unique $x \in X$ such that ¤$(x) = x$.

(b) State Inverse function theorem and implicit function theorem.

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PG–941	**MMS–17**

M.Sc. DEGREE EXAMINATION —
JUNE, 2010.

First Year

Mathematics

COMPLEX ANALYSIS AND NUMERICAL ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

161. An analytic function in a region Ω whose modulus is constant, reduce to a constant. Prove this.

162. State and prove Weierstrass theorem on essential singularity.

163. Discuss Schwarz lemma.

164. By theory of residues evaluate $\int_0^\infty \frac{\cos x}{x^2 + a^2} dx$.

165. Using bisection method, find a root of the equation $x^3 - 4x - 9 = 0$ correct to three decimal places.

166. Using Gauss-Jordan method, solve the system of equations.

$x_1 + 2x_2 + x_3 = 5$, $2x_1 + 3x_2 - x_3 = 7$, $2x_1 - x_2 + 3x_3 = 12$.

167. Given that $u_{16} = 39$, $u_{18} = 85$, $u_{22} = 151$, $u_{24} = 264$ and $u_{26} = 388$. Compute u_{20} using the difference table.

168. Using Taylor series method, solve the problem $y' = 2x + 3y$, $y(0) = 1$, to get $y(0.4)$ with length $h = 0.1$.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

169. If $u(x, y)$ and $v(x, y)$ have first-order partial derivatives that satisfy the Cauchy-Riemann equations, show that $f(z) = u + iv$ is analytic.

170. Let $f(z)$ be analytic in a domain D and $f'(z) \neq 0$ in D. Prove that $w = f(z)$ is a conformal mapping.

171. State and prove Cauchy's residue theorem.

172. If $f(z)$ is meromorphic in Ω with the zeros a_j and poles b_k, prove that

$\frac{1}{2\text{®}\, i} \int_{\text{¥}} \frac{f'(z)}{f(z)} dz = \sum_j n(\text{¥}, a_j) - \sum_k n(\text{¥}, b_k)$ for every cycle ¥ which is homologous to zero in Ω.

173. Using triangulization method, solve the system of equations.

$$2x - 3y + 10z = 3$$
$$-x + 4y + 2z = 20$$
$$5x + 2y + z = -12.$$

174. Evaluate $\int_0^1 \frac{dx}{1+x}$ using composite Simpson's rule with 2, 4 and 8 equal subintervals.

175. Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval $[0,1]$. Use the fourth order classical Runge-Kutta method.

176. Calculate the n^{th} divided difference of $\frac{1}{x}$.

PG–942 **MMS–18**

M.Sc. DEGREE EXAMINATION — JUNE, 2010.

First Year

Mathematics

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

177. Define :

(a) Mathematical Expectation and

(b) Moment generating function of a random variable X.

178. Give the density function of (a) Negative Binomial distribution and (b) Chi-square distribution.

179. Enumerate the different types of sampling.

180. What is an unbiased estimator? Give an example.

181. Give the confidence interval for estimating (a) variance (b) difference of two proportions of a sample.

182. Define (a) most powerful test and (b) uniformly most powerful test.

183. Explain SPRT.

184. State Rao Blackwell theorem and give its application.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

185. State and prove Chebychev's inequality.

186. Suppose that a two-dimensional continuous random variable (X,Y) has joint p.d.f. given by $f(x,y)=\begin{cases} 6x^2y; 0<x<1 \\ \quad\quad 0<y<1 \\ 0, \quad \text{elsewhere} \end{cases}$

(a) Verify that $\int_0^1 \int_0^1 f(x,y)\,dxdy = 1$.

(b) Find $P(0<X<\frac{3}{4}, \frac{1}{3}<Y<2)$, $P(X+Y<1)$, $P(X>Y)$ and $P(X<1/Y<2)$.

187. A box contains 'a' white and 'b' black balls. 'c' balls are drawn at random. Find the expected value of the number of white balls drawn.

188. Derive the distribution of $\overline{X}$ and $\frac{ns^2}{\pm^2}$.

189. Derive the confidence interval for estimating the mean of a sample.

190. State and prove Neymann-Pearson theorem.

191. State and prove Rao-Cramer inequality.

192. Prove that every most powerful or uniformly most powerful critical region is necessarily unbiased.

PG–943 **MMS–25**

M.Sc. DEGREE EXAMINATION –
JUNE, 2010

Second Year

Mathematics

TOPOLOGY AND FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

193. Let S be a subbasis for topology. Prove that the collection of all finite intersection of elements of S is a basis for a topology.

194. Let A be a subset of the topological space X. Let A' be the set of all limit points of A. Prove that $\overline{A} = A \cup A'$.

195. Let X and Y be topological spaces. Let $f : X \to Y$ be a homomorphism. Prove that $f(U)$ is open if and only if U is open.

196. Prove that a path connected space is connected.

197. Show that a subspace of a second countable space is second countable.

198. State and prove the uniform boundedness theorem.

199. If M and N are closed linear subspaces of a Hilbert space H such that $M \perp N$ then prove that the linear subspace $M + N$ is closed.

200. Prove that $\|T^{*}T\| = \|T\|^{2}$, where $T \to T^{*}$ is adjoint operation on $B(H)$.

SECTION B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

201. Let X be a set and be a basis for a topology T on X. Prove that T equals the collection of all union of elements of .

202. Let Y be a subspace of X and A be a subset of X. Let $\overline{A}$ denote the closure of A in X. Prove that the closure of A in Y equals $\overline{A} \cap Y$.

203. Let $f : X \to Y$ be a function. Prove that the following are equivalent.

(a) f is continuous,

(b) for $A \subset X$, $f(\overline{A}) \subset \overline{f(A)}$

(c) for every closed set B in Y, $f^{-1}(B)$ is closed in X.

204. Prove that every closed subset Y of a compact space X is compact.

205. State and prove Tietze extension theorem.

206. Discuss the open mapping theorem.

207. State and prove Bessel's inequality.

208. Let H be a Hilbert space and let f be an arbitrary functional in H^*. Prove that there exists a unique vector y in H such that $f(x) = (x, y)$ for every x in H.

PG–944 **MMS–26**

M.Sc. DEGREE EXAMINATION – JUNE, 2010

Second Year

Mathematics

OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

209. Rewrite in standard form :

Maximize $Z = 5x_1 + 2x_2 + x_3$

Subject to the constraints
$$2x_1 + x_2 - x_3 = 6$$
$$x_1 + x_2 + x_3 \leq 8,$$
$$x_1, x_2, x_3 \geq 0.$$

210. What is linear goal progrmming?

211. Draw a network for a project consisting of jobs $A - I$ such that $A < D$, $A < E$, $B < F$, $D < F$, $C < G$, $C < H$, $F < I$, $G < I$.

212. Define probabilistic programming.

213. Give the standard form of a pure integer programming problem.

214. Solve the following game :

	B_1	B_2
A_1	8	–3
A_2	–3	1

215. Write a note on. Pure birth and Pure death queuing models.

216. Find the stationary point of $f(x) = x^4 + x^2$.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

217. Solve by Simplex method :

Maximize $Z = 2x_1 + 2x_2 + 4x_3$

Subject to the constants $2x_1 + 3x_2 + x_3 \leq 300$

$x_1 + x_2 + 3x_3 \leq 300$,

$x_1 + 3x_2 + x_3 \leq 240$,

$x_1, x_2, x_3 \geq 0$.

218. Using dual simplex method solve :

Minimize $Z = x_1 + 2x_2 + 3x_3$

Subject to the constraints $x_1 - x_2 + x_3 \geq 4$,

$x_1 + x_2 + 2x_3 \leq 8$,

$x_2 - x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$.

219. What are the assumptions of the minimum cost flow problem? Give the network representation of the problem.

220. If $p_1 + p_2 + ... + p_n = 1$, show that

$z = p_1 \log p_1 + p_2 \log p_2 + ...p_n \log p_n (p_i \geq 0)$ is

minimum when $p_1 = p_2 = ... = p_n = 1/n$. (Use Dynamic Programming Method).

221. Use LPP method to solve the game with pay-off matrix .

$$\begin{bmatrix} -1 & 2 & 1 \\ 2 & -2 & 2 \\ 3 & 3 & -3 \end{bmatrix}.$$

222. Use branch and bound method to solve the following integer LPP;

Maximize $z = 2x_1 + 2x_2$

Subject to the constraints $5x_1 + 3x_2 \leq 8$,

$x_1 + 2x_2 \leq 4$,

$x_1, x_2 \geq 0$ and are integers.

223. Trucks arrive at a loading dock at an average rate of 4 trucks/hours. The loading of a truck takes 10 minutes on the average by a crew of 3 loadmen. The operating cost of a truck is Rs. 20/ hr and loadmen are paid Rs. 6 each per hour. Is it advisable to add another crew of 3 loadmen?

224. Solve the following game using Dominance property.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{pmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix} \end{array}$$

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PG–945	**MMS-27**

M.Sc. DEGREE EXAMINATION –
JUNE 2010.

Second Year

Mathematics

GRAPH THEORY AND ALGORITHMS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

225. Prove that a graph G is connected if and only if for every partition of V into nonempty sets V_1, and V_2, there is an edge with one end in V_1 and one end in V_2.

226. Show that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

227. Define connectivity and edge-connectivity of a graph and illustrate with example.

228. Prove that a connected graph has an Euler trail if and only if has at most two vertices of odd degree.

229. Prove that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in the minimum covering.

230. Show that a non-empty graph G is 2-colourable if and only if G is bipartite.

231. Prove that the chromatic polynomial of a tree of order p is $k(k-1)^{p-1}$.

232. State and prove Euler's formula on connected graph.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

233. Show that a graph is bipartite if and only if it contains no odd cycle.

234. Write the Kruskal's Algorithm and prove that it results a minimum weighted spanning tree.

235. State and prove the Haval and Hakimi theorem.

236. Show that a connected graph G is Eulerian if and only if each vertex of G has even degree.

237. State and prove the Tutte's theorem.

238. State and prove Vizing theorem to find the bounds for the edge chromatic number.

239. Show that if G is 4-chromatic then G contains a subdivision K_4.

240. State and prove the five colour theorem.

M.Sc. DEGREE EXAMINATION — JUNE, 2010.

Second Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

241. Let ¤$_1$, ¤$_2$ be solutions of a linear homogeneous equation with constant coefficients $L(y)=0$ on an interval I containing a point x_0 . Prove that $W(¤_1, ¤_2)(x)=e^{-a_1(x-x_0)} \cdot W(¤_1, ¤_2)(x_0)$.

242. Find a particular solution of $y''+y=\operatorname{cosec} x$.

243. For any n prove that $P_n(-x)=(-1)^n P_n(x)$ and hence $P_n(-1)=(-1)^n$.

244. Let Φ be a fundamental matrix of the system of equations $y'=A(x)y$ such that $\Phi(0)=E$ where A is a constant matrix. Prove that $\Phi(t+s)=\Phi(t)\,\Phi(s)$ for all $t,s\in I$.

245. Let $f(x)$ be periodic with period μ . Let A be an $n\times n$ constant matrix. Prove that a solution of $y'=Ay+f(x)$ is periodic of period μ if and only if $y(0)=y(\mu)$.

246. Find the solution of the equation $\dfrac{\partial^2 z}{\partial x^2}-\dfrac{\partial^2 z}{\partial y^2}=0$.

247. Prove that $\cdot\,(x,y,z)=\dfrac{q}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}$.

satisfies Laplace equation.

248. Define equipotential surfaces and also Green's function.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

249. Discuss the variation of parameter method to find a particular solution of non homogeneous equation $L(y)=b(x)$.

250. Discuss the solution of the Legendre equation $(1-x^2)y''-2xy'+\sim(\sim+1)y=0$ where $\sim$ is a constant.

251. Compute the indicial polynomial and their roots for the equation $4x^2y''+(4x^4-5x)y'+(x^2+2)y=0$.

252. Let A be a constant matrix. Prove that the general solution of the system of equations $y'=Ay$ on interval I is $y(x)=e^{xA}c$. Further show that the solution of the system with the initial condition $y(x_0)=y_0$ is $y(x)=e^{(x-x_0)A}\cdot y_0,\ x_0\in I$.

253. Let f be continuous on $(-\infty,\infty)$ and periodic with period μ. Prove that a necessary and sufficient condition for the system $y'=Ay+f(x)$ to have a unique periodic solution with period μ is that the system $y'=Ay$ has no nonzero periodic solution of period μ.

254. State and prove Picard's theorem.

255. Prove that a function ¤ is a solution of the initial value. Problem $y'=f(x,y),\ y(x_0)=y_0$ on an interval I containing the point x_0 if and only if ¤ is a solution of the integral equation $y=y_0+\int_{x_0}^{x} f(t,y)dt.$

256. State and prove Kelvin's inversion theorem.