

# 050(E)

(JULY, 2009)

*Time : 3.00 Hours]*

*[Maximum Marks : 100*

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## **Instructions :**

1. Answer **all** the questions.
2. Write your answers according to the instructions given below with the questions.
3. Begin each **section** on a **new page**.

## **SECTION - A**

*Given below are 1 to 15 multiple choice questions. Each carries **one** mark. Write the letter (A), (B), (C) or (D) in your answer book of the alternative which you feel is the correct answer of the questions.*

1. The origin be shifted to  $(-2, 3)$  so that the new co-ordinates of ..... would be  $(3, -2)$ .  
(A)  $(-1, 1)$  (B)  $(1, 1)$   
(C)  $(1, -1)$  (D)  $(-1, -1)$
2. For all  $a, b, c \in \mathbb{R}$ ,  $2a + 3b + 5c = 0$ , the line  $ax + by + c = 0$  passes through fixed point ..... ( $a^2 + b^2 \neq 0$ )  
(A)  $(2, 3)$  (B)  $(-2, -3)$   
(C)  $\left(\frac{-2}{5}, \frac{-3}{5}\right)$  (D)  $\left(\frac{2}{5}, \frac{3}{5}\right)$
3. Circle  $x^2 + y^2 - 2ax - 2ay + a^2 = 0$ ,  $a \neq 0$  .....  
(A) passes through origin. (B) touches only X-axis.  
(C) touches only Y-axis. (D) touches both the axes.

4. One of the end point of the focal chord of Parabola  $y^2 = 16x$  is  $\left(\frac{1}{4}, 2\right)$ , then the other end point is .....
- (A)  $\left(2, \frac{1}{4}\right)$  (B)  $\left(\frac{1}{4}, -2\right)$   
 (C)  $(64, -32)$  (D)  $(-64, 32)$
5. Equation of a tangent to  $\frac{x^2}{3} - \frac{y^2}{2} = 1$  and parallel to  $y = x$  is .....
- (A)  $x - y + 1 = 0$  (B)  $x + y - 1 = 0$   
 (C)  $x - y + 2 = 0$  (D)  $x + y + 2 = 0$
6. If  $|\bar{x}| = |\bar{y}| = |\bar{x} - \bar{y}|$ , then  $|\bar{x} + \bar{y}| = \dots\dots\dots$
- (A)  $\sqrt{3} \bar{x}$  (B)  $\sqrt{3} |\bar{x}|$   
 (C)  $3 \bar{x}$  (D)  $3 |\bar{x}|$
7. For a parallelogram ABCD,  $\vec{AB} = \bar{a}$  and  $\vec{BC} = \bar{b}$ , then its area = .....
- (A)  $\frac{1}{2} |\bar{a} \times \bar{b}|$  (B)  $|\bar{a} \times \bar{b}|$   
 (C)  $|(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})|$  (D) None of these
8. A plane cuts axes at A, B, C such that the centroid of  $\Delta ABC$  is  $(1, 3, 1)$ , the equation of this plane is .....
- (A)  $\frac{x}{3} + \frac{y}{1} + \frac{z}{3} = 3$  (B)  $\frac{x}{1} + \frac{y}{3} + \frac{z}{1} = 3$   
 (C)  $3x + 3y + z = 3$  (D) None of these
9.  $x \in \mathbb{N}^* (-2, \delta) \Rightarrow f(x) \in \mathbb{N}(9, 0.01)$ , then the maximum value of  $\delta$  is ....., where  $f(x) = 5 - 2x$ .
- (A) 0.001 (B) 0.005  
 (C) 0.009 (D) None of these

10. If  $\frac{d}{dx} f(x) = g(x)$ , then  $\frac{d}{dx} \left( -\frac{1}{f(x)} \right) = \dots\dots\dots$

(A)  $\frac{-1}{(f(x))^2}$

(B)  $\frac{1}{(f(x))^2}$

(C)  $\frac{-f(x)}{(g(x))^2}$

(D)  $\frac{g(x)}{(f(x))^2}$

11.  $\int \{ \sin(\log x) + \cos(\log x) \} dx = \dots\dots\dots + c.$

(A)  $x \sin(\log x)$

(B)  $x \cos(\log x)$

(C)  $\sin(\log x)$

(D)  $\cos(\log x)$

12.  $\int \frac{1}{\sqrt{\left(\log \frac{1}{2}\right)^2 - x^2}} dx = \dots\dots\dots + c.$

(A)  $\sin^{-1} \left( \frac{x}{\log \frac{1}{2}} \right)$

(B)  $-\sin^{-1} \left( \frac{x}{\log 2} \right)$

(C)  $\sin^{-1} \left( \frac{x}{\log 2} \right)$

(D) None of these

13.  $\int_0^{2a} \frac{f(x)}{f(x) + f(2a-x)} dx = \dots\dots\dots$

(A)  $a$

(B)  $-a$

(C)  $\frac{a}{2}$

(D)  $\frac{-a}{2}$

14. Degree of a differential equation  $\left( \frac{d^2 y}{dx^2} \right)^{\frac{2}{3}} = \left( y + \frac{dy}{dx} \right)^{\frac{1}{2}}$  is  $\dots\dots\dots$

(A) 1

(B) 2

(C) 3

(D) 4

15. A particle is projected vertically upward with a velocity of 24.5 m/sec., then velocity of that particle after 2 sec. is ..... m/sec.
- (A) 4.9 (B) -4.9  
(C) -14.7 (D) 14.7

**SECTION - B**

*Answer the following 16 to 30 questions. Each carries one mark.*

16. Find the incentre of the triangle whose vertices are  $(\sqrt{3}, 1)$ ,  $(0, 0)$ ,  $(0, 2)$ .
17. Obtain the location of point  $(a \cos \alpha, a \sin \alpha)$  in the plane relative to a circle  $x^2 + y^2 = r^2$ , where  $\alpha \in (-\pi, \pi]$ ,  $|a| < r$ ,  $a \neq 0$ .
18. There is a point on the Parabola  $y^2 = 8x$  whose Y- coordinate is two times the X- coordinate. If this point is not the vertex of the Parabola, find that point.
19. Let L and L' be the feet of perpendicular drawn from the foci S and S' respectively to the tangent at any point P(x, y) of the ellipse  $\frac{x^2}{9} + \frac{y^2}{16} = 1$ , then find SL · S'L' .

**OR**

Find the measure of eccentric angle of point  $(-2, -2\sqrt{2})$  on the ellipse  $2x^2 + y^2 = 16$ .

20. If  $\alpha, \beta, \gamma$  are the direction angles of the vector  $\vec{r}$ , then find the value of  $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$  .
21. Force  $2\vec{i} + 2\vec{j} + 2\vec{k}$  is applied at B(1, 2, 3); find the torque around A(-1, 2, 0).
22. Find the equation of the line through (4, 3, 2) and parallel to the line  $\frac{x-10}{15} = \frac{y-2}{5} = \frac{z-1}{3}$ .
23. If the position vectors of the end points of a diameter of a sphere are  $4\vec{i}$  and  $2\vec{j}$ , find the Cartesian equation of the Sphere.

24. The formula connecting the periodic time  $T$  and length  $l$  of a pendulum is  $T = 2\pi\sqrt{l/g}$ . If there is an error of 2% in measuring the length  $l$ , what will be the percentage error in  $T$ ?

25. Discuss the validity of Rolle's Theorem for  $f(x) = x^{\frac{1}{4}}$ ,  $x \in [-1, 1]$ .

**OR**

The radius of a right circular cone is constant. If there is an error  $\delta h$  in measuring its height, what will be the error in measurement of its volume?

26. Evaluate :  $\int \frac{e^x - 1}{e^x + 1} dx$ .

27. Obtain the value of  $\int_0^{\pi} \sin^3 x \cdot \cos^3 x dx$ .

**OR**

If  $\int_n^{n+1} f(x) dx = n^3$ , then find the value of  $\int_{-3}^3 f(x) dx$ .

28. Obtain the differential equation representing the family of curves  $y = a \cos^{-1} x + b$ , where  $a$  and  $b$  are arbitrary constants.

29. A body projected in vertical direction attains maximum height 16 m. Find its initial velocity.

30. Range of a projectile is  $\frac{4}{\sqrt{3}}$  times its maximum height  $\frac{u^2 \sin^2 \alpha}{2g}$ . Find measure of angle of projection.

### SECTION - C

*Answer the following questions from 31 to 40.*

*Each carries TWO marks, as directed in the question.*

31. The equation of a perpendicular bisector of  $\overline{AB}$  is  $5x + 2y - 18 = 0$ , if  $A$  is  $(-3, 2)$ ; then find the co-ordinates of the midpoint of  $\overline{AB}$ .

**OR**

Find the co-ordinates of the foot of the perpendicular from  $A(a, 0)$  to the line

$$y = mx + \frac{a}{m}; m \neq 0.$$

32. Find the locus of point P such that the slopes of the tangents drawn from P to a Parabola have (i) constant sum (ii) constant non zero product.

**OR**

Find the co-ordinates of the points of contact of the tangents drawn from (1, 5) to the Parabola  $y^2 = 24x$ .

33. If the difference between measures of the eccentric angles of P and Q is  $\frac{\pi}{2}$  and

if  $\overleftrightarrow{PQ}$  cuts intercepts  $c$  and  $d$  on the axes, prove that  $\frac{a^2}{c^2} + \frac{b^2}{d^2} = 2$ .

34. Find the equation of a curve from every point of which the tangents to the Hyperbola  $\frac{x^2}{144} - \frac{y^2}{36} = 1$  intersect at right angles.

**OR**

If the chord of the Hyperbola joining P( $\alpha$ ) and Q( $\beta$ ) on the hyperbola subtends a right angle at the centre C(0, 0); prove that  $a^2 + b^2 \sin \alpha \cdot \sin \beta = 0$ .

35. If  $\bar{a} \neq \bar{0}$ ,  $\bar{b} + \bar{c} \neq \bar{0}$  and  $\bar{a} + \bar{b} + \bar{c} \neq \bar{0}$ ; show that  $\bar{a}$ ,  $\bar{b} + \bar{c}$ ,  $\bar{a} + \bar{b} + \bar{c}$  are coplanar.

36. The dot product with  $\bar{i} + \bar{j} + \bar{k}$  of the unit vector having the same direction as the vector sum of  $2\bar{i} + 4\bar{j} - 5\bar{k}$  and  $\lambda \bar{i} + 2\bar{j} + 3\bar{k}$  is 1, find  $\lambda$ .

37. Find the equation of the sphere passing through the point O(0, 0, 0), A(-a, b, c), B(a, -b, c), C(a, b, -c).

38. If  $y = \tan^{-1}\left(\frac{3-2x}{2+3x}\right)$ , then find  $\frac{dy}{dx}$ .

**OR**

If  $y = (\cos^{-1} x)^2$ , then prove that  $(1-x^2)y_2 - xy_1 = 2$ .

39. Obtain the intervals in which function  $f(x) = x^3 - 6x^2 - 36x + 2$  in increasing and decreasing.

40. Evaluate :  $\int_0^1 x^2(1-x)^{\frac{1}{2}} dx$ .

**SECTION - D**

Answer the following questions from 41 to 50, each carrying **THREE** marks as directed in the question.

41. If A is  $(-2, 1)$  and B is  $(1, -7)$ ; find a point on  $\overleftrightarrow{AB}$  such that  $5AP = 3AB$ .

**OR**

If  $P(at^2, 2at)$ ,  $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$  and  $S(a, 0)$  are three points, show that  $\frac{1}{SP} + \frac{1}{SQ}$  is independent of  $t$ .

42. Find the co-ordinates of points which are at minimum and maximum distance from the point  $(-7, 2)$  on the circle  $x^2 + y^2 - 10x - 14y - 151 = 0$ .

**OR**

Find the equation of the circle that touches the Y-axis and passes through  $(-2, 1)$  and  $(-4, 3)$ .

43. Prove by using vectors that the perpendicular bisectors of the sides of a triangle are concurrent.

44. Find the measure of the angle between two lines if their direction cosines  $l, m, n$  satisfy  $l + m + n = 0$ ,  $l^2 - m^2 + n^2 = 0$ .

45. Obtain the foot of perpendicular, perpendicular distance and equation of perpendicular line from  $A(2, 3, 2)$  on  $\vec{r} \cdot (1, -2, 1) = -5$ .

46. Find :  $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{16r^2 + 8r - 3}$ .

47. By using mean value theorem for  $\log(1+x)$  in  $[0, x]$ , prove that

$$0 < \frac{1}{\log(1+x)} - \frac{1}{x} < 1, \text{ where } x > 0.$$

**OR**

The slope of the tangent at the point  $(1, 1)$  on the curve  $xy + ax + by = 2$  is 2, find  $a$  and  $b$ .

48. Evaluate:  $\int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx.$

49. Evaluate:  $\int \frac{6x+7}{\sqrt{(4-x)(5-x)}} dx \quad (x < 4).$

50. Solve:  $\frac{dy}{dx} + \frac{4xy}{x^2+1} = \frac{1}{(x^2+1)^2}.$

**SECTION - E**

*Answer the following questions from 51 to 54, each carrying FIVE marks.*

51. The lines  $x - 2y + 2 = 0$ ,  $3x - y + 6 = 0$  and  $x - y = 0$  contain the three sides of a triangle. Determine the co-ordinates of the orthocentre without finding the co-ordinates of the vertices of the triangle.

**OR**

Find the equation of the line passing through  $(\sqrt{3}, -1)$  if its perpendicular distance from the origin is  $\sqrt{2}$ .

52. Find:  $\lim_{x \rightarrow 1} \left\{ \frac{25}{x^{25} - 1} - \frac{15}{x^{15} - 1} \right\}.$

53. If  $f(x + y) = f(x) \cdot f(y)$ , then find  $f'(3)$ ; where  $f(x) = \log(e + x)$ ,  $x > 0$ .

54. Evaluate:  $\int_0^1 \sin^{-1} \left( \frac{2x}{1+x^2} \right) dx.$

**OR**

Prove that the area of the region bounded by the circle  $x^2 + y^2 = 16$  and the Parabola  $y^2 = 6x$  is  $\frac{4}{3} (4\pi + \sqrt{3})$ .

**050(E)**  
**(MARCH, 2009)**

**Time : 3.00 Hours]**

**[Maximum Marks : 100**

**Instructions :**

1. All the questions are **compulsory**.
2. Write your answers according to the instructions given below with the questions.
3. Begin each section from a *new* page.

**SECTION - A**

Questions from 1 to 15 are multiple choice questions, each carrying **ONE** mark.  
Write the letter of the correct option (A), (B), (C) or (D) in your answer book from the alternatives.

**15**

1. Measure of the angle between lines  $x = 1$  and  $\sqrt{3}x + y - 4 = 0$  is .....  
(A)  $\frac{2\pi}{3}$  (B)  $\frac{\pi}{6}$   
(C)  $\frac{7\pi}{6}$  (D)  $\frac{\pi}{2}$
2. If the lines  $5x - ky - 7 = 0$  and  $2x + 3y + 5 = 0$  are mutually perpendicular to each other, then  $k = \dots\dots$   
(A)  $-\frac{10}{3}$  (B)  $\frac{10}{3}$   
(C)  $-\frac{15}{2}$  (D)  $\frac{15}{2}$
3. The equation of the line passing through the diametric points of the circle  $(x - 2)^2 + (y - 3)^2 = 25$  is ....  
(A)  $2x + y = 5$  (B)  $(x - 2) + (y - 3) = 25$   
(C)  $x + y = 5$  (D)  $x + 2y = 10$
4. Parametric equation of  $y^2 = 12x$  are ..... ( $t \in \mathbb{R}$ ).  
(A)  $(6t, 3t^2)$  (B)  $(4t^2, 3t)$   
(C)  $(4t^2, 8t)$  (D)  $(3t^2, 6t)$

5. Measure of the angle between asymptotes of  $x^2 - y^2 = 1$  is .....
- (A)  $\frac{\pi}{4}$  (B)  $\frac{\pi}{3}$   
 (C)  $\frac{\pi}{2}$  (D) 0
6. If  $\bar{x} = (2, 0)$ ,  $\bar{y} = (3, 0)$ , then  $|\bar{x} + \bar{y}|$  .....  $|\bar{x}| + |\bar{y}|$ .
- (A) = (B) <  
 (C) > (D) None of them.
7. Projection of  $\bar{i}$  in the direction of  $\bar{j}$  is .....
- (A)  $\bar{o}$  (B)  $\bar{i}$   
 (C)  $\bar{j}$  (D)  $\bar{k}$
8. If the directions of  $\frac{x-1}{c} = \frac{y+2}{-2} = \frac{z-3}{4}$  and  $\frac{x-5}{1} = \frac{y-3}{1} = \frac{z+1}{c}$  are same, then  $c =$  .....
- (A) -2 (B) 2  
 (C) 4 (D) -4
9. The radius of the Sphere  $x^2 + y^2 + z^2 - 2x - 2y - 2z - 1 = 0$  is .....
- (A) 4 (B)  $\sqrt{2}$   
 (C) 2 (D)  $\sqrt{13}$
10.  $\lim_{x \rightarrow \infty} x(\sqrt[3]{2} - 1) =$  .....
- (A)  $\log_2 x$  (B)  $\log_2 e$   
 (C)  $\log_e 2$  (D)  $\log_e x$
11. If  $x = 3t^2 - 6t + 5$  and  $v = 0$ , then  $t =$  .....
- (A) 0 (B) 1  
 (C) 5 (D) -1
12. The rate of increasing volume of the Sphere w.r.t., its surface area is .....
- (A)  $2r$  (B)  $r/2$   
 (C)  $r/4$  (D)  $r/3$

13.  $\int \left( \sin \frac{x}{2} + \cos \frac{x}{2} \right)^2 dx = \dots\dots\dots + c.$

- (A)  $(x - \cos x)$  (B)  $(x + \sin x)$   
 (C)  $(x + \cos x)$  (D)  $(x - \sin x)$

14. The area of the region bounded by  $y = \tan x$ , X axis,  $x = 0$  and  $x = \frac{\pi}{4}$  is ..... units.

- (A)  $2 \log 2$  (B)  $\frac{1}{2} \log 2$   
 (C)  $\log 2$  (D)  $1$

15. The order and degree of the differential equation  $\sqrt[3]{\left(\frac{d^2y}{dx^2}\right)^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

are .....

- (A)  $4, 4$  (B)  $2, 4$   
 (C)  $4, 2$  (D)  $2, 2$

**SECTION - B**

*Answer the following questions from S.Nos. 16 to 30.*

*Each question carries **ONE** mark.*

**15**

16. Find the point P(a, b) on the line, whose parametric equations are  $x = 2t + 1$  and  $y = 1 - t$ ,  $t \in \mathbb{R}$ , so that  $a + b = 1$ .

17. Find the Cartesian equation of the Circle whose parametric equations are  $x = -1 + 2 \sin \theta$  and  $y = 1 + 2 \cos \theta$ ,  $\theta \in (-\pi, \pi)$ .

18. For the Parabola  $y^2 = -8x$ , obtain length of the Latus rectum and the end points of latus rectum.

19. Find the measure of eccentric angle for Ellipse

$$\frac{x^2}{100} + \frac{y^2}{25} = 1 \text{ at } (-6, 4) \text{ which belongs to it.}$$

**OR**

Find the equation of the Ellipse, whose vertex is  $(\pm 5, 0)$  and foci  $(\pm 4, 0)$ .

20. Find direction angles of  $\bar{x} = (3, 0, -4)$ .
21. Decide the direction for vectors  $\bar{x} = (1, 3, 1)$ ,  $\bar{y} = (2, 6, -2)$  whether same, opposite or different.
22. Find the displacement of a particle  $3\bar{i} + 2\bar{j} - 5\bar{k}$  due to the force  $2\bar{i} - \bar{j} - \bar{k}$  and find the work done.
23. Find the perpendicular distance from P(1, 2, 3) to line

$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$$

24. Obtain the equation of the Plane passing through (3, 4, 2), (2, 2, -1) and (7, 0, 6).

25. Obtain  $\frac{d}{dx}(x^x)$ .

26. Find :  $\frac{d}{dx}\left(\sin^{-1} \frac{x}{a}\right)$  where  $a \neq 0$ .

27. Find :  $\int_{-1}^1 \sin^3 x \cos^4 x dx$ .

**OR**

Find the value of  $\int_0^{\sqrt{2}} \sqrt{2-x^2} dx$ .

28. Obtain  $\int [e^{a \log x} + e^{x \log a}] dx$ .

29. Find the area of the region bounded by curve  $x = y$ , X-axis and the lines  $x = 2$  and  $x = 3$ .

30. Find the differential equation for the family of the lines  $y = mx + c$ , where  $m, c$  are arbitrary constant.

### SECTION - C

Answer the following questions from Nos. 31 to 40.

Each question carries **TWO** marks. Answer as directed in the questions.

**20**

31. Two of the vertices of a triangle are  $(1, -6)$  and  $(-5, 2)$ . The centroid of the triangle is  $(-2, 1)$ . Find the third vertex of the triangle. Also find the area of this triangle.

**OR**

Prove that not both co-ordinates of all the vertices of an equilateral triangle can be rational numbers.

32. Prove that  $y = x + 3$  is a tangent to the Parabola  $y^2 = 12x$ . Also find co-ordinates of its point of contact.
33. If the difference between measures of the eccentric angles of P and Q is  $\frac{\pi}{2}$  and if  $\overleftrightarrow{PQ}$  cuts intercepts  $c$  and  $d$  on the axes,

then prove that  $\frac{a^2}{c^2} + \frac{b^2}{d^2} = 2$ , where P and Q are on the Ellipse.

34. For the rectangular Hyperbola  $x^2 - y^2 = 9$ , consider the tangent at  $(5, 4)$ . Find the area of the triangle, which this tangent makes with two asymptotes.
35. Find the point of intersection of the lines

$$L: \frac{x-3}{1} = \frac{y+2}{-1} = \frac{z+1}{-1} \quad \text{and} \quad M: \frac{x}{2} = \frac{z+3}{3}; y = -1.$$

36. Get the radius and the centre of the Circle that is formed by the intersection of Sphere  $x^2 + y^2 + z^2 = 25$  and the Plane  $2x + 2y + z = 12$ .

37. For  $y = (\tan x)^x + x^{\tan x}$ , find  $\frac{dy}{dx}$ .

38. Curves  $x^2y = 1$  and  $a^5y = x^3$  are intersecting perpendicularly. Prove that  $a^6 = 6$ .

39. If initial velocity of Projectile is 28 m/s. and horizontal range is 40 m., then find measure of angle of Projection.

**OR**

When acceleration is constant and instantaneous speed is 22 m/s, the particle covers a distance of 10320 m. in 60 seconds, then find the acceleration.

40. Obtain  $\int \tan^3 x \, dx$ .

**OR**

Obtain  $\int \frac{x + \sin x}{1 + \cos x} \, dx$ .

**SECTION - D**

*Answer the following questions from S.Nos. 41 to 50.*

*Each question carries **THREE** marks. Answer as directed in the questions. 30*

41. For A(1, 2) and B(2, 1), find the point dividing  $\overline{AB}$  from A's side in  $n$  equal parts. From this, find the coordinates of the point of trisection.
42. Find the equation of the Circle which passes through (1, -2) and (4, -3) and having a centre lies on the line  $3x + 4y = 7$ .

**OR**

Find the equation of the Circle, if common chord of the circles

$x^2 + y^2 - 4x = 0$  and  $x^2 + y^2 - 6y = 0$  as the diameter.

43. Find a unit vector in  $\mathbb{R}^3$  making an angle of measure  $\frac{\pi}{3}$  with each of the vector (1, -1, 0) and (0, 1, 1).
44. Prove that centroid and incentre for the equilateral triangle are same.  
With this, if A(6, 4, 6), B(12, 4, 0) and C(4, 2, -2), then find the incentre for triangle ABC by using vectors.

45. Find the foot of the perpendicular and equation of perpendicular line passing through  $(2, -1, 2)$  to plane  $2x - 3y + 4z = 44$ .

OR

Find the equation of the Plane passing through the line of intersection of the planes  $3x - 4y + 5z = 10$  and  $2x + 2y - 3z = 4$  and parallel to the line  $x = 2y = 3z$ .

46. A water tank is in the form of an inverted cone. The radius and the height of the cone are 10 cm. and 20 cm. respectively. If water is poured in the tank at the rate of  $5 \text{ cm}^3 / \text{second}$ , then how fast the height of the water will be increasing when its depth is 15 cm. ?

OR

Prove that the length of the part of the tangent intercepted between the axes coordinate of the Curve  $x^{2/3} + y^{2/3} = a^{2/3}$  is constant. ( $a > 0$ )

47. Evaluate :  $\int_{-1}^2 |2x-1| dx$ .

48. Find the area of the region bounded by curves  $y = x^2$  and  $y = x + 2$ .

OR

Find the area of the region bounded by curves  $y^2 = 4x$  and  $x^2 = 4y$ .

49. Evaluate  $\frac{dy}{dx} + \frac{2y}{x} = e^x$ .

50. Find  $\lim_{x \rightarrow 1} \frac{a^x - (a+1)^x + 1}{x-1}$ .

**SECTION - E**

Answer the following questions from S.Nos. 51 to 54.

Each question carries **FIVE** marks.

**20**

**51.** Find the equation of a circum-circle of the triangle formed by the lines

$$x + y = 6, \quad 2x + y = 4 \quad \text{and} \quad x + 2y = 5.$$

**OR**

Find the co-ordinates C and D for the square ABCD,

if A(-1, 3) and B(2, -2).

**52.** Find :  $\lim_{x \rightarrow -1^+} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x+1}}$

**53.** If  $2x = y^{1/m} + y^{-1/m}$  ;  $x \geq 1$ ,

then prove that  $(x^2 - 1)y_2 + xy_1 = m^2 y$ .

**54.** Evaluate :

$$\int \cos 2x \cdot \cos 4x \cdot \cos 6x \, dx.$$

**OR**

Evaluate :

$$\int (4x+3)\sqrt{x^2 - 4x + 13} \, dx.$$