

**B.C.A. II Semester Examination, March 2011**  
**MATHEMATICS**

Time : 3 Hours

Max. Marks : 80

- Instructions :** 1) Answer **all** questions in Part A, **6** out of **8** questions in Part B, and **3** out of **5** questions in Part C.  
2) Part A : Questions from 1 to 8 carry 1 mark and 9 to 14 carry 2 marks each.  
3) Part B : Each question carries 5 marks.  
4) Part C : Each question carries 10 marks.

PART – A

1. If  $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$  find AB.
2. If  $(G, *)$  is a group,  $a, b, c, d \in G$  then find  $(ab^{-1}c^{-1}d)^{-1}$ .
3. Find the centre of the circle  $3x^2 + 3y^2 - 6x + 4y + 1 = 0$ .
4. Find the  $n^{\text{th}}$  derivative of  $e^{mx}$ .
5. Evaluate  $\int_0^{\pi/2} \cos^8 x \, dx$ .
6. Find the value of  $\int \frac{1}{e^x} dx$ .
7. Find the integrating factor for  $x^2 \frac{dy}{dx} + xy = \frac{1}{1-x}$ .
8. State Cayley-Hamilton Theorem.

**P.T.O.**

9. Find  $x$  if  $[x \ 2] \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 2$ .
10. If  $x^2 + y^2 = a^2$  find  $\frac{dy}{dx}$ .
11. Find  $\frac{dy}{dx}$  if  $y = \tan^{-1} \sqrt{\frac{1 - \cos x}{1 + \cos x}}$ .
12. Evaluate  $\int e^x (\sin x + \cos x) dx$ .
13. Solve by Cramer's rule :  $2x - 3y = 5$ ,  $7x - y = 8$ .
14. Test for exactness and solve,  $(x - xy^2)dx + (3y - x^2y) dy = 0$ .

### PART – B

Answer **6** questions. **Each** question carries **5** marks.

1. Find the adjoint of the matrix  $A = \begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$  and hence find  $A^{-1}$ .
2. Find the unit vector perpendicular to each of the vectors  $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$  and  $\vec{b} = 3\hat{i} + 4\hat{j} + \hat{k}$
3. Find the equation of the plane passing through a point  $(1, 2, 1)$  and contains the line  $\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{1}$ .
4. Find the equation of the tangent plane to the sphere  $x^2 + y^2 + z^2 - 6x - 4y - 2z + 5 = 0$  at the point  $(1, 0, 2)$ .

5. Evaluate  $\lim_{x \rightarrow b} \frac{x^b - b^x}{x^x - b^b}$ .

6. Find the  $n^{\text{th}}$  derivative of  $\frac{x}{(x-1)^2(x+2)}$ .

7. Evaluate  $\int \frac{x+2}{x(x^2+1)} dx$ .

8. Evaluate  $\int \frac{x+2}{\sqrt{4x-x^2}} dx$ .

### PART – C

1. Solve using matrix method  $x - y - 2z = 3$  ;  $2x + y + z = 5$  and  $4x - y - 2z = 11$ .

2. Prove that the set  $\mathbb{Z}$  of integers is an abelian group under the binary operation defined by  $a * b = a + b + 1$  for all  $a, b \in \mathbb{Z}$ .

3. Show that the angle between the two lines whose direction cosines satisfy the relations  $l + m + n = 0$  and  $2lm + 2nl - mn = 0$  is  $\pi/3$ .

4. Evaluate  $\int (x+1) \sqrt{\frac{x+2}{x-2}} dx$ .

5. Solve :  $(x^2 + 2xy - y^2) dx + (y^2 + 2xy - x^2) dy = 0$ .

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