UG–743	BMS-01

B.Sc. DEGREE EXAMINATION – JUNE, 2006.

First Year

Mathematics

CALCULUS AND CLASSICAL ALGEBRA

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Differentiate $y = (\sin x)^x$.

2. Find the equation of the tangent to the curve $y = \dfrac{6x}{x^2 - 1}$ at the point $(2,4)$.

3. Find the envelope of the family of straight lines $y = mx + \dfrac{a}{m}$ for different values of m.

4. Using Bernoulli's formula, find $\int (2x^2 + 1)\cos x dx$.

5. If $u_1 + u_2 + ... + u_n + ...$ is convergent then show that $\lim_{n \to \infty} u_n = 0$.

6. Test the convergence of the series $\sum_{n=0}^{\infty} \frac{n^3 + 1}{2^n + 1}$.

7. Define absolute convergence and conditional convergence.

8. Show that $\sqrt{x^2 + 16} - \sqrt{x^2 + 9} = \frac{7}{2x}$ nearly for sufficiently large values of x.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. (a) If $y = \sin(m \sin^{-1} x)$, prove that $(1 - x^2) y_2 - x y_1 + m^2 y = 0$.

(b) Find $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$. (7 + 3)

10. (a) What is the radius of curvature of the curve $x^4 + y^4 = 2$ at the print $(1,1)$?

(b) Find the pedal equation of the curve $r = ae^{\theta \cot \alpha}$. (6 + 4)

11. Obtain a reduction formula for $\int \sin^n x$, where n is a positive integer. Deduce a formula to evaluate $\int_0^{\pi/2} \sin^n x \, dx$. (6 + 4)

12. Express $F(x) = x^2$ as a Fourier series with period 2π, to be valid in the interval $-\pi$ to π. (10)

13. Show that the series $\frac{1}{1^k} + \frac{1}{2^k} + \frac{1}{3^k} + ...$ is convergent when $k > 1$ and divergent when $k \leq 1$. (10)

14. (a) State Leibuitz test for checking the convergence of an alternating series.

(b) Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{2n - 1}.$$ (3 + 7)

15. Sum the series

$$\frac{1^2}{1!} + \frac{1^2 + 2^2}{2!} + \frac{1^2 + 2^2 + 3^2}{3!} + ... + \frac{1^2 + 2^2 + ... + n^2}{n!} + ...$$

(10)

16. Show that if $x > 0$,

$$\log x = \frac{x - 1}{x + 1} + \frac{1}{2} \frac{x^2 - 1}{(x + 1)^2} + \frac{1}{3} \frac{x^3 - 1}{(x + 1)^3} + ...$$ (10)

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UG–744 **BMS-02**

B.Sc. DEGREE EXAMINATION – JUNE, 2006.

First Year

Mathematics

TRIGONOMETRY, ANALYTICAL GEOMETRY OF THREE DIMENSIONS AND VECTOR CALCULUS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

1. Show that $\cos 6\theta = 1 - 18\sin^2\theta + 48\sin^4\theta - 32\sin^6\theta$.

2. Prove that $\cosh^2 x - \sinh^2 x = 1$.

3. Find the real and imaginary parts of Log $(a + ib)$.

4. Find the equation of the line joining the points (2,3,7) and (2,−5,8).

5. Find the centre and radius of the sphere

$$x^2+y^2+z^2-2x-4y+6z-11=0.$$

6. Find the equation of the cone of the second degree which passes through the axes.

7. Find the divergence of $x^2\,\bar{i}+y^2\,\bar{j}+z^2\,\bar{k}$.

8. If $\bar{F}=3x^2y\,\bar{i}+(x^3-3y^2)\,\bar{j}$, compute $\int \bar{F}\cdot d\bar{r}$ along $y^2=4x$ from (0,0) to (4,4).

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

Each question carries 10 marks.

9. Prove that $\cos^5\theta\cdot\sin^4\theta=\frac{1}{2^8}[\cos 9\theta+\cos 7\theta-4\cos 5\theta-4\cos 3\theta+6\cos\theta]$.

10. If $\cos(x+iy)=\cos\theta+i\sin\theta$, show that $\cos 2x+\cosh 2y=2$.

11. Sum the series upto n terms:

$$\tan^{-1}\frac{4}{4.1^2+3}+\tan^{-1}\frac{4}{4.2^2+3}+\tan^{-1}\frac{4}{4.3^2+3}+....$$

12. Prove that the lines $\frac{x+1}{-3}=\frac{y+10}{8}=\frac{z-}{2}$; $\frac{x+3}{-4}=\frac{y+1}{7}=\frac{z-4}{1}$ are coplanar. Find also the point of intersection and the plane through them.

13. Find the shortest distance between the lines $\frac{x+3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. Also find the equation of the line of shortest distance.

14. Find the equation of the sphere which passes through the point (1,−2,3) and the circle $z=0$, $x^2+y^2+z^2-9=0$.

15. Find $\nabla\cdot\bar{F}$ and $\nabla\times\bar{F}$ of the vector point function $\bar{F}=xz^3\,\bar{i}-2x^2yz\,\bar{j}+2yz^4\,\bar{k}$ at the point (1,−1,1).

16. Evaluate $\iint_S \bar{F}\cdot\hat{n}\,ds$ where $\bar{F}=4xz\,\bar{i}-y^2\,\bar{j}+yz\,\bar{k}$ and S is the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$.

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UG–745 **BMS-03**

B.Sc. DEGREE EXAMINATION – JUNE 2006.

First Year

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Solve : $xp^2 - 2yp + x = 0$.

2. Solve : $(D^2 + 3D + 2)\ y = x^2$.

3. Solve : $(D^2 - 2D + 4)\ y = e^x . \cos x$.

4. Form a partial differential equation by eliminating the function of ϕ from

$$\phi\,(x + y + z,\ x^2 + y^2 - z^2) = 0.$$

5. Solve : $(mz - ny)p + (nx - lz)\ q = ly - mx$.

6. Solve : $z = px + qy + c\sqrt{1 + p^2 + q^2}$.

7. Prove that $L\,[t^n] = \dfrac{\Gamma(n+1)}{s^{n+1}}$ and hence find $L\,[t^{\frac{1}{2}}]$.

8. Find $L^{-1}\left[\dfrac{s+2}{(s^2 + 4s + 5)^2}\right]$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Solve : $(2x+1)^2\, y'' - 2(2x+1)y' - 12y = 6x$.

10. Solve : $(4D+2)x + (9D+31)y = e^t$

$(3D+1)x + (7D+24)y = 3$.

11. Solve : $\dfrac{d^2y}{dx^2} + 4y = \operatorname{cosec}(2x)$ by the method of variation of parameters.

12. Verify the condition of integrability in the equation $(y+z)\,dx + (z+x)\,dy + (x+y)\,dz = 0$ and solve it.

13. Solve : $p + 3q = 5z + \tan(y - 3x)$.

14. Solve : $p^3 + q^3 = 27z$.

15. (a) Find $L[(f(t)]$ where

$f(t) = 0$ when $0 < t < 2$

$= 3$ when $t > 2$.

(b) Find $L^{-1}\left[\dfrac{s^2}{(s-1)^3}\right]$.

16. Using Laplace transform, solve the equation $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} - 3y = \sin t$ given that $y = \dfrac{dy}{dt} = 0$ when $t = 0$.
